ВОПРОСЫ ТЕОРИИ

Optimal Budget and Seigniorage Targeting Policy in a Transition Economy

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Targeting policy is treated as a marginal capping of seigniorage and government expenditures, respectively. Appropriate policies of stabilization might be performed by the government rather independently due to existence of a distorted and asymmetric financial market in a transition economy. Feasible strategies are represented as solutions to the Bellman equation in the optimal stopping problem for stochastic processes of budget expenditures and government borrowing on the open market. Respective options to stop spending and borrowing prescribe the optimal policy for budget expenditures as well as for seigniorage targeting. Implementation of such a policy is, in essence, an imposition of the call provisions on the government debt, the optimal value of which is equal to the value of the opportunity to borrow at the optimal point.

Russian economy during its transition towards market has experienced periods of high inflation and depression, followed by a prolonged stagnation complicated with huge arrears. In fact, these persistent arrears *a la Russe* are not just «accounts receivable» but nothing less than bad debts in disguise, initiated by the government itself in its ultimately futile efforts to win in the uncompromized tag-of-war against inflation, inspired to a great extent by IMF. The fight against inflation, as it was appeared, had required the government to violate the basic contract - to pay on its obligations. Transformation of arrears into the major and permanent characteristics of Russian economic landscape inevitably led to a barter which accounted now of approximately 40 percent of all transactions made in Russian economy. The seemingly endless sequence of severe cuts of budget expenditures, on the fiscal side, and sharp imbalance of money supply and demand for money, on the monetary side, became possible due to barter, hence lack of competition and, in their turn, contributed to the prolonged depression.

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In efforts to finance its budget deficit, or better to say, the debt service component of the deficit, while trying to curb inflation, Russian government started in 1994 to borrow heavily on the open market. In just four years, 1994-1998, the internal government debt has grown by factor of $7 \cdot 10$ making the debt service ratio, both internal and external, jumping up almost to 40 per cent of current budget expenditures. Nevertheless, the skyrocketing debt could not help to overcome production slump that had taken place in 1991-1996 and amounted to no less than 30 per cent of 1990 GDP level. An evident inability of the government to revive domestic production was mainly due to the fact that all the money it had raised on the open market were spent to finance its current external and internal liabilities neglecting completely the necessity to support of plummeting investment demand. On the other hand, persistent depression and unemployment has appeared to be major factors that impeded the mere search of nonmonetary means of debt servicing and stabilization.

The government seems to confine itself within the vicious circle. Persistent depression and ineffectiveness of the system of tax collection had forced it to borrow heavily on the open market. In order to meet financial obligations the government had to sustain its expenditures in excess of taxes that, in its turn, leads to heavy borrowing, budget deficit and further accumulation of debt. Its efforts to cut the budget, though being helpful to some extent in tackling the problem of debt servicing, inevitably provoked further depression followed by the sharp decline in current consumption and profound social inequality.

The culmination of the government financial distress has taken place in August 1998 when a combination of a shrinking positive balance of current account, due to decreasing world oil prices, together with nonresident investors walking out from the Russian government bond market gave rise to an *avalanche* of outgoing dollars from Russian economy that proved to be unbearable for international reserves of the central bank. In futile efforts to save fragile bond market the government rose the bond yield up to 120-150 percent annually, but in atmosphere of uncertainty such actions resulted only in a debt default *de facto*, that came into effect after the official announcement of the scheme of the debt restructuring. The latter was followed by the rouble flotation accompanied with wild volatility of a currency market, and subsequent deep depreciation of the rouble. The gloomy picture of Russian economy should be added with two strokes: virtual collapse of the banking system that has given rise to an additional impetus to inflation, hitherto partially hidden by arrears (bad debts); and to the renewed decline in the domestic production, this time initiated in the deeply hit export oriented sectors.

As it seems, sharply diminished or even lost possibility to borrow, both internally and externally, have largely contributed to an increase in Russian economic difficulties in the short run. Hence it is worthwhile to make some theoretical comments as to underline the importance of a possibility to borrow. In our opinion, such an option cannot be taken as granted *a priori*, but should be considered as being contingent on the confidence of lenders which, in its turn, is solidly based upon the proper fulfillment by the borrower of his contract obligations, or on meticulous debt service, in this particular context.

Debt stabilization: a simple deterministic model

First, the inevitability of the debt default in a stagnant economy is a well known fact that easy to analyze and foresee using a simple model of the growing debt. Let $b = \frac{B}{Y}$ be a ratio of debt, B, in nominal terms to nominal product or nominal GDP, Y. The instantaneous rate of the debt growth then will be equal to

$$\dot{b} = \frac{\dot{B}}{Y} - \alpha b, \qquad (1)$$

where $\alpha = \frac{\dot{Y}}{Y}$ is the instantaneous rate of growth of nominal GDP. If the budget deficit is financed solely with borrowing on the open market, then it might be represented by the following equation:

$$\dot{B} = iB + (G - T) = iB + \bar{d}, \qquad (2)$$

where i is the constant continuously compounded nominal rate of interest, $\bar{d} = \frac{G - T}{Y}$ is the primary deficit ratio to nominal GDP, and G, T are nominal government expenditures and taxes, respectively.

By substituting (2) into (1) we get a simple ordinary differential equation for the debt ratio $b(t)$:

$$\dot{b} = (i - \alpha)b + \bar{d}, \qquad (3)$$

which for given initial ratio $b(0) = b_0$ and constant rate of primary deficit has a solution:

$$b(t) = [b_0 + \frac{1}{i - \alpha}\bar{d}]\exp[(i - \alpha)t] - \frac{1}{i - \alpha}\bar{d}. \qquad (4)$$

In the general case of the positive difference, $(i - \alpha) > 0$, that is, for nominal interest rate larger than the rate of nominal GDP growth, solution (4) would be unstable. The only way to stabilize solution (4) is to make difference $(i - \alpha)$ nonpositive by stimulating the economy to grow. It should be noted, however, that even for the distorted and crippled system of taxation the primary deficit in contemporary Russian economy was fully financed, meaning that $\bar{d} = 0$, the fact that will be fully exploited later.

Though being rather obvious, the model (3) is too simple, and it disguises several important features of the actual process of the debt accumulation and service. The most important is the fact that budget deficit is normally financed through combination of borrowing and seigniorage issuance that makes the government to be better off while postponing, if possible, the debt stabilization. Situation of August, 1998 in Russian economy that was created by the announcement of the debt restructuring, in fact government debt default, vividly illuminated the important point: the mere possibility of borrowing is a valuable option. If granted, it reflects the continued faith of the financial community that even under the most trying conditions the government will honor its obligations. Thus, our second conclusion is that the importance of borrowing should not be underestimated and being taken as granted under any circumstances. It is a valuable one, hence the option to borrow has to have a nonzero value. Otherwise, the cost of debt service would be much higher, and these opportunity costs form the value of such an option.

Budget stabilization: an overview

Explicit analysis of budget and debt stabilization problem in a specific set-up of the economy in transition makes it clear that, theoretically at least, solution to this problem should be viewed as a simultaneous seigniorage and expenditures capping. Meanwhile, as it is well known, being taken together they might give rise to a depression. But whether in fact fiscal and monetary factors are so rigidly tied up in the process of debt servicing? And what kind of *the sustainable budget policy* implementation is required for in particular conditions of a transition economy? Is it, at last, worthwhile to stop borrowing and cap government expenditures, and when is it optimal to implement such a policy?

It is the almost unanimously accepted fact that no government tackling the problem of debt service would neglect the possibility of considering seigniorage as a potential source of budget deficit financing. The stimuli to produce seigniorage in a *«high inflation economy»* [1] are different from that of in a *«low inflation»* economy, though. Transition economy, in our opinion, being very similar to the economy of *«high inflation»* [2], do possess with some peculiarities that make interconnections between real and financial markets to be very loose and ambiguous. In fact, the ambiguity is so high that it gives rise to the emergence of the actual separability between real and financial markets in a transition economy. Financial markets are just emerging in the process of transition, and their influence on the development of real market and upon resources allocation is generally very limited. Virtual inelasticity of investment demand in respect to changes in interest rate, both real and nominal, might be taken as an example of existence of weak trade-offs between portfolio investments, real capital accumulation and development of the real market in the period of transition. On the other hand, the actual separability of real and financial markets in the process of economic transition creates necessary conditions for the government to behave in a more flexible manner rather than it would have been possible in a market economy.

The model being developed in this paper will show these additional possibilities existing for the government in transition period that helps it to operate more or less independently on real and financial markets. Given taxes, seigniorage issuance becomes the only source of debt service, and the model demonstrates feasibility of the debt stabilization policy that might be implemented simultaneously with balancing of the budget. High degree of uncertainty that is characteristic for the process of economic transition plays the crucial role in fulfillment of this rigid requirement without triggering inflation nor undermining the prospects of economic growth.

Our analysis of the budget stabilization problem is based to a large degree on results presented by *G. Bertola and A. Drazen* [3]. They had shown the existence of the modified trade-offs between current consumption and future government expenditures in an uncertain environment, and the necessity to implement a policy of budget cutting from time to time. In the model developed in this paper their results were modified in some respects. The debt in real terms is treated as a variable influenced by fiscal and monetary factors simultaneously. The problem of sustainable debt policy implementation is described along the lines of *budget capping policy* rather than *budget cuts policy*. We use different from *the Bertola-Drazen's* hypothesis for modelling the random process of government expenditures, namely of geometric Brownian movement without drift, that enables us to avoid a rather uncomfortable feature of the spending negativity.

The Bertola-Drazen analysis of the budget stabilization is generalized to the problem of simultaneous stabilization of the debt and budget. In order to find a solution in a closed form, the latter is represented as the «optimal stopping» problem in dynamic programming for Ito's stochastic processes. Methodology of solving such problems was developed and considered as a standard technique in financial economics, especially in the contingent claims analysis. In this respect we borrowed a lot from books by *J. Ingersoll* [4] and by *A. Dixit* and *R. Pindyck* [5]. In respect of presentation of a stabilization process as a problem of option pricing, the importance of a paper by *M. Miller and L. Zang* [6] should be stressed, though in our model inflation does not appear as an explicit driving force but, rather as a limiting factor influenced implicitly by the behavior of a government and private investors approaching, the optimal point (reflecting barrier).

Private sector behavior in a transition economy

The problem of debt and budget stabilization is studied in a context of stochastic behavior of a private sector and a government in a transition economy.

The *representative private consumer* is assumed to exist infinitely and to behave rationally in the sense that it is pursuing maximum expected utility of private consumption over the entire future[2)]. The expectation of future consumption, $\mathrm{E}_t\{U[C(t+\tau)]\}$ is discounted by the riskless rate of return on government bonds that supposed to be known and fixed, $r > 0$. Hence the criterion for the private sector behavior might be written as a functional:

$$\max \mathrm{E}_t \int_t^{\infty} U[C(\tau)] \exp[-r(\tau - t)] d\tau \;. \tag{6}$$

In the present context of Russian economic conditions an assumption of a stagnant production, $Y(t) = Y = const$, seems to be plausible enough, implying that

2) It is reasonable to assume the government as well as the particular economy to constitute an infinitely living entity, though sometimes, fortunately rarely enough, it might collapse like the USSR in 1991. For the contemporary Russian economy an assumption of risklessness of government bonds might be considered as a vulnerable one, especially after August, 1998.

private sector allocates the yield on the wealth and constant real output among consumption, $C(t)$, taxes, $T(t)$,and the wealth accumulation, $\dot{A}(t)$, (all in real terms) for the future perspective subject to the constraint:

(7) $$\mathrm{E}_t \int_t^{\infty} [C(\tau) + T(\tau) - Y] \exp[-r(\tau - t)] d\tau = A(t) .$$

Functional (6) is maximized subject to the constraint (7), and both functionals are assumed to be convergent. For this isoperimetric variational problem, the shadow price of the unit of private consumption is fixed, $\lambda(\tau) = \lambda = const$, the Lagrange integrand does not depend on marginal consumption, and the Euler-Lagrange equation $L_C - \frac{d}{d\tau} L_{C'} = 0$ takes a simple form of

(8) $$\mathrm{E}_t U'[C(\tau)] = \lambda ,$$

to which the optimal path for consumption should suffice.

Assuming private sector to be linear risk tolerant, or existence of quadratic utility function, we get from (8) constant rational expectations of private consumption $\mathrm{E}_t\{C(\tau - t)\} = C(t)$ for all $\tau \geq t$. That helps us to transform the constraint (7) into the following equation:

(9) $$C(t) = Y + rA(t) - r\int_t^{\infty} \mathrm{E}_t\{T(\tau)\} \exp[-r(\tau - t)] d\tau .$$

Usage of more convenient ratios to GDP of all the variables involved, while assuming constant output, helps us to arrive to a simple equation among consumption, assets and taxes:

(10) $$c(t) = 1 + ra(t) - rv(t) .$$

Equation (10) that was received and analyzed in the Bertola-Drazen paper [3] shows the parametric dependence of the optimal path for the private consumption on the expected present value of the future discounted stream of government taxes due to existence of a (convergent) functional

(11) $$v(t) = \int_t^{\infty} \mathrm{E}_t\{h(\tau)\} \exp[-r(\tau - t)] d\tau ,$$

where $h(\tau)$ is the ratio of real taxes to real GDP.

The government behavior in a transition economy

The government behavior in a transition is represented as a mathematical model of a standard budget financing process in the real terms:

$$h + s + \dot{b} = g + rb \,. \tag{12}$$

The left hand side of the consolidated government (and the central bank) balance sheet discloses major sources of financing in real terms: taxes, h, seigniorage, $s \equiv \frac{\dot{M}}{P} = \dot{m} + pm$, and new debt or borrowing, $\dot{b}$, while the right hand side of (12) consists of current expenditures, g, and the debt service, rb. In the balance sheet seigniorage represents the net change in the money supply as a result of open market operations of the central bank with government bonds.

Table 1.

Balance sheet of deficit financing

ASSETS	LIABILITIES
h	g
$\dot{b}$	rb
s	

On the RHS of Table 1, the government optimizes an option to stop its spending thus capping the budget expenditures on the optimal level, $g = g^*$. On the LHS, the government takes the option to stop borrowing on the open market thus implementing the strategy of *«the seigniorage targeting»*. Hence the same equation (12) is used in both cases for the explicit definition of the appropriate government policy, though under different assumptions in respect to taxes, expenditures and seigniorage. In a solution of the capital budgeting problem seigniorage is supposed to be equal to zero, $s = 0$, thus reducing it to a problem of fiscal regulation. Once the optimal value for the government expenditures is found, it is used as an exogenous constant to determine taxes that serve as a parameter in the equation for the optimal consumer behaviour (10). Then assuming the feasibility of the balanced budget (without debt servicing) the policy of a «seigniorage targeting» might be implemented being aimed at the debt stabilization by exercising the option «to stop borrowing».

The government in transition period is supposed to behave rationally, the assertion that is represented by *the sustainable budget and debt policy* equation:

$$b(t) = \int_t^{\infty} \mathrm{E}_t\{s(\tau) - [g(\tau) - h(\tau)]\} \exp[-r(\tau - t)]d\tau \,. \tag{13}$$

The market value of a debt, $b(t)$, is equal to the expected value of the accumulation of net differences between future flows of seigniorage, $s(\tau)$, and taxes, $h(\tau)$, on the one hand, that are used as means of financing government expenditures, $g(\tau)$, and debt service, rb, on the other hand, in accordance with the balance sheet being analyzed above. Due to the random nature of the variables involved, the debt constraint in the integral form (13) thus represents a solution (existed under some assumptions) to the standard stochastic debt equation:

$$\frac{1}{dt}\mathrm{E}_t[db] = rb - [s - (g - h)]. \quad (14)$$

As it well known, equation (14), in its turn, is a real term equivalent of the standard budget equation in nominal terms (taken in absolute magnitudes):

$$\mathrm{E}_t\{\frac{\dot{M}}{P}\} + \mathrm{E}_t\{\dot{b}\} = (G - T) + (R - p)b,$$

that is just the formal representation of the above sited balance sheet. It is assumed for the latter that seigniorage either exceeds real deficit:

$$S \equiv \frac{\dot{M}}{P} > (G - T),$$

or just a positive value for zero primary budget deficit, $G - T = 0$.

There are two aspects in analysis of the ordinary differential equation (14) that need some comments in the context of the problem under consideration. The first is concerned with specification of boundary conditions. Assuming for simplicity deterministic process of a debt accumulation with $s = 0$, $b(0) = b_0$, and $(h - g) = const$, we easily get for equation (12) with $r > 0$ and $b_0 - [(h - g)/r] \neq 0$, the general solution:

$$b(t) = [b_0 - \frac{h-g}{r}]\exp(rt) + \frac{h-g}{r}, \quad (15)$$

that proves to be unstable[3]. The present discounted value of a debt is represented in (15) as a constant equilibrium value, and nonzero initial discrepancy between the face and present values would grow up or diminish indefinitely in the future. In the case of equality between face and present values the particular solution to (12) is just the constant perpetuity of the future excess taxes. Thus solution to a debt service problem in a growing economy would depend upon a relation between rates of growth of a debt and of the real GDP, as it was shown earlier.

The alternative approach to the solution of (12) has been proposed by *T.Sargent and N.Wallace* (see *S.Turnovsky* [7] for the concise exposition of their approach). The initial conditions are not specified but a transversality condition:

$$\lim_{\tau \to \infty} \mathrm{E}_t\{s(\tau) - [g(\tau) - h(\tau)]\} \exp[-r(\tau - t)] = 0, \quad (16)$$

is imposed instead that allows for the existence of a stable solution (13) to equation (14). In the deterministic case similar to the analyzed above, we simply get the stable particular solution for (12), or the present value of a debt in the form

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[3] Solution (15) is a slightly modified formula (4) for $\alpha = 0$, *and* $-\bar{d} = h - g$.

$$\int_0^{\infty}(h-g)\exp[-r\tau]d\tau=\frac{h-g}{r}$$

without any reference to initial conditions.

Another comment concerns interpretation of a debt as a mixed, fiscal and monetary, variable. It should be noted, that the ratio of government debt in real terms to GDP, $b(t)$, at any moment of time is, by definition, the ratio of budget deficit, $[g(\tau)-h(\tau)]$, continuously compounded for every $\tau \le t$ and accumulated over the infinite period in the past:

$$b(t)=\int_{-\infty}^{t}[g(\tau)-h(\tau)]\exp[r(t-\tau)]d\tau, \tag{17}$$

which is for $t=0$ is the face value of a debt $b(0)=b_0$, and hence (11) is considered as a fiscal variable. In accordance with *the Ricardian equivalence* the market value of government bonds is equal to the expected present discounted stream of the future (for $\tau \ge t$) excesses of taxes over government expenditures (in their respective ratios to GDP):

$$b(t)=\int_{t}^{\infty}\mathrm{E}_t\{h(\tau)-g(\tau)\}\exp[-r(\tau-t)]d\tau. \tag{18}$$

Equation (18) asserts *the sustainable budget policy constraint* which shows that the expected present value of a debt is paid off only through the collection of taxes in excess of government expenditures. It should be stressed however, that in our opinion, it is more preferable to use the sustainable debt constraint (13) instead of the budget constraint (18). Rationale for such substitution is that no government coping with the problem of the debt service does practically neglect the possibilities of money issuance, at least that is true for the economy in transition. Debt as the expected present value of future payments should be studied as a forward variable that reflects the influence of both monetary and fiscal factors, the seigniorage issuance in particular, and (18) might be viewed as a special form of constraint (13).

Capping of diffusion process of government expenditures

Now substitute the sustainable debt policy constraint (13) into equations (10) and (11). By doing it we get *the general equation of private consumption dynamics:*

$$c(t)=1+ra(t)-r[b(t)+\int_{t}^{\infty}\mathrm{E}_t\{g(\tau)-s(\tau)\}\exp[-r(\tau-t)]d\tau], \tag{19}$$

which is of major interest for our analysis, and will be studied later under successful assumptions: $s(\tau)=0$, and $g(\tau)-h(\tau)=0$.

Let us study, first, the problem of capping of budget expenditures. Since $s = 0$, the market value of government debt is considered as a pure fiscal variable. In accordance with (13) the government issues no seigniorage, and in order to pay out the debt it has to collect taxes in excess of its current expenditures. This alternative actually deals entirely with the government policy of sustaining its balanced budget and might be studied along the way of the *Bertola- Drazen* approach.

Within this hypothesis equation (19) might be transformed into the following system:

$$c(t) = 1 + r[a(t) - b(t)] - rv(t)\,, \tag{20}$$

where

$$v(t) = \int_t^\infty E_t\{g(\tau)\} \exp[-r(\tau - t)]d\tau\ . \tag{21}$$

Equation (21) reveals the dependence of current consumption not only upon the yield on the net foreign assets, $r[a(t) - b(t)]$, but upon the expected present value of the stream of government expenditures in the future. For $g(\tau)$ being deterministic, equation (20) might be treated along the simple reasoning of Ricardian equivalence: expectations of future growth in government expenditures give rise to a contraction in the present consumption. Rational agents in this case start to reassess immediately the prospects of the higher tax leverage in the future that will inevitably follow the growing government expenditures, and in order to offset unfavorable outcomes are getting started to economize on their current consumption.

For $g(\tau)$ being the stochastic variable there appears no straightforward procedure for evaluation of the future stream of government expenditures. More of that, with the probability approaching to one, the unrestricted random process $g(\tau)$ as $\tau \rightarrow \infty$ would cause the process $v(t)$ to exceed the unity in equation (14), and there is no other way to prevent such an outcome except as *to cap* the process of expenditures. In economic terms when amount of budget expenditures reach some threshold it gives the government no other alternative as to fix or hack out its spending. The thorny and highly painful procedure of a budget cut (or *the sequester* as it is called in contemporary Russian budget practice) involves prolonged, heated and frustrated political debates, and produces heavy economic and social losses upon its exercise.

Returning to equations (20) and (21), it should be noted that expectations of future taxes and expenditures are not observable as such. In order to obtain a relation between observable consumption and future government spending, it is necessary to specify a process for the latter and to find how expectations of its present discounted value depend on its current observable values. It is a well known fact that economic development in transition is a highly uncertain process where information condensed to a very high degree and past experience quickly become obsolete. Government does not control all the variables that influence the process of its spending, nor even know their future values.

These considerations make plausible the hypothesis of government expenditures as a random process with independently and identically distributed (i.i.d.) increments. In this model it is assumed for simplicity that changes in government spending do not lead to changes in expectations of their movement in the future, i.e.

$$\mathrm{E}_t\{dg\} = 0\,,$$

where dg is the i.i.d. increments of government spending. The random process of government expenditures follows hypothesis of *a geometric Brownian motion* without drift[4], namely

(22) $$dg = \sigma g dW\,,$$

where σ is a parameter of variance in logs of government expenditures, and $W(t), t \geq 0$ is *a standard Wiener process* [8], or a Gaussian random variable with

$$\mathrm{E}\{W(t)\} = 0,\ Var\{W(t)\} = t,\ Cov\{W(s),\, W(t)\} = \min(s,t)\,.$$

Rational government under this simple assumption expects no changes in the level of its expenditures for all future perspective $\tau \geq t$: $\mathrm{E}_t\{g(\tau - t)\} = g(t)$. The simple quality of this process makes possible its representation in the following form:

$$g(\tau - t) = g(t)\exp[\sigma W(\tau - t)]\,.$$

The budget capping policy

For the stochastic process of government expenditures (22) it is necessary to answer the question as to where it is optimal to stop spending. Actually, the government while increasing its spending always retain the policy of capping the budget later in its arsenal, provided to be free to choose the precise moment of implementation of such a policy. This possibility should in general be attributed to the ambiguity in the public assessment of the exact amount of the threshold expenditures g^*. Due to such an uncertainty, the government always has a possibility to wait as to when precisely to change its policy by getting started to implement budget restrictions. In other words, it seems reasonable to assume that all the way along the spending trajectory the government holds *an option of «budget capping later»*.

Let the value of a policy «to delay budget capping» be $f(g)$. The policy of the postponement of the budget capping obeys the inequality

(23) $$f(g) \geq v(g) - \hat{z}\,,$$

where $\hat{z}$ are cumulative costs, in terms of production slump and unemployment, that would have been incurred by the budget capping. Once budget capping is exer-

[4] Smoothing of the tax rates by the government makes tax rates a random walk. This gets the principle applied to the case of seigniorage that implies that nominal interest rates and inflation should be smoothed as well and that such smoothing makes these series approximately random walks [9, 10].

cised, it would inevitably lead to economic and social losses, the present value of which is assumed to be a constant ratio to GDP, $\hat{z}$. It should be noted that the value of the proposed strategy, $f(g)$, has to be added to the total costs, once the policy has changed, alias the option exercised. Hence value of the proposed strategy should be either strictly positive, or equal to zero, depending on the relative proportion of costs of capping to the benefits of spending continuation.

Rational behavior of the government for the continuation region, $g < g^*$, follows the strict inequality:

$$f(g) > v(g) - \hat{z}.$$

In other words, for all the values $g < g^*$, benefits of spending (the expected present value of government expenditures), $v(g)$, are less than costs of stopping plus value of the abandoned strategy, $\hat{z} + f(g)$, making thus unoptimal to cap the budget in this region. There exists an optimal point for the process specified above [5]. In this point the government *«exercises its option»* to cap expenditures, getting instead the expected present value of its budget spending $v(g^*)$ and thus admitting all the costs involved, or:

$$f(g^*) = v(g^*) - \hat{z}, \text{ at the optimal point, } g = g^*.$$

It follows from the differential form of (21) that until some, *a priori* unknown, threshold g^* is reached, the government would continue to increase its spending along the path described by the stochastic differential equation

$$\frac{1}{dt}\mathrm{E}_t\{dv\} = rv - g. \quad (24)$$

Equation (24) says, that market effectiveness of the expected present value of government expenditures should be equal to their current value plus increase in their capital value. In accordance to such *a «no-arbitrage condition»* stochastic differential equation (24) has (21) as its solution represented by function $v(t,g) = v(g)$ for every fixed t. Hence behavior of a government in the continuation region, $g < g^*$, and in the optimal point g^* might be described by *the Bellman equation*:

$$v(g) = \max\{\frac{1}{r}[g + \frac{1}{dt}\mathrm{E}_t\{dv\}],\ f(g^*) + \hat{z}\}. \quad (25)$$

Government expenditures are stopped to increase in the stationary point of equation (24). When the government performs budget capping, its expenditures that are being hacked out, become worthless and their marginal value goes to zero, hence the following condition takes place:

$$\lim_{g \to g^*} \frac{dv}{dg} = 0. \quad (26)$$

The point that satisfies to this condition is called *the reflecting point* of the random process $g(t)$ [11]: when the process reaches this point, it is reflected back into the inner part of the interval $[0, g^*]$.

Solution to the budget capping problem

It is possible to find a closed form solution for the process described by (22), (25) and (26). Let us calculate expectations of changes for stochastic process of government expenditures during infinitesimally short period, dt, by applying Ito's lemma to the process $v(g)$:

$$\mathrm{E}_t\{dv\} = \mathrm{E}_t\{v'(g)dg + \frac{1}{2}v''(g)(dg)^2\} = \frac{1}{2}\sigma^2 g^2 v''(g)dt .$$

It can be transformed using (24) into following linear second order differential equation in respect to $v(g)$, which is sometimes referred to as *the Black-Scholes equation*:

(27) $$\frac{1}{2}\sigma^2 g^2 v''(g) - rv(g) + g = 0 .$$

The particular solution to this equation is given by function $v(g) = \frac{1}{r}g$, meaning that government in order to get present discounted value of its expenditures simply capitalizes its current value by factor $\frac{1}{r}$ while imposing no restrictions upon this process.

The general solution to the homogeneous part of (27) is given by

(28) $$v(g) = K_1 g^{\beta_1} + K_2 g^{\beta_2} ,$$

where $\beta_1 > 1, \beta_2 < 0$ are characteristic roots of the quadratic

(29) $$\frac{1}{2}\sigma^2 \beta(\beta - 1) - r = 0 .$$

According economic sense, when government expenditures go negligibly small, $g \to 0$, their present expected value should go to the zero too, $\lim_{g \to 0} v(g) = 0$, and for these reasons $K_2 = 0$. In other words, the origin appears to serve as *the absorbing point* for the process of government expenditures[5)].

Consequently, economically conceivable solution to equation (27) takes the form of a function

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[5)] For the Bertola-Drazen assumption [1] of a simple Brownian motion with drift, constant K_2 goes to zero when government expenditures go to $-\infty$, situation which seems to be at odds with economic reality.

(30) $$v(g) = Kg^{\beta} + \frac{1}{r} g ,$$

where $K \equiv K_1, \beta \equiv \beta_1 > 1$. The present expected value of budget spending is subject to regulation, so the constant in (30) reflects nonzero probability of expenditures capping at some moment in the future, and for that reason it should be negative, $K < 0$, as it will be shown later.

As to the option of *«budget capping»,* the government is supposed to expect the increase of its value over the infinitesimally short intervals of time for all continuation region, $g < g^*$, up to the reflecting barrier, $g = g^*$, where it exercises the option to wait:

(31) $$f(g) = \max\{\frac{1}{rdt} \mathrm{E}_t[df], v(g^*) - \hat{z}\} .$$

Using for the option evaluation the same reasoning as for *«the primitive asset»* (government expenditures in this particular case), we arrive to the equation:

(32) $$\frac{1}{2} \sigma^2 g^2 f''(g) - rf(g) = 0 .$$

Equation (32) differs from (27) in one respect : it is homogeneous because an option does not bring current yield to its holder but delivers capital gain only upon its exercising. The option has the same point of absorption, $f(0) = 0$, hence equation (32) has a solution

(33) $$f(g) = Lg^{\beta}, \text{ where } \quad L \equiv L_1, \ \beta \equiv \beta_1 > 1 .$$

The optimal point of budget capping

As it is known from dynamic programming, at the reflecting barrier the process should satisfy simultaneously to *the value matching condition*:

(34) $$v(g^*) = f(g^*) + \hat{z} ,$$

and to *the smooth pasting condition*:

(35) $$v'(g^*) = f'(g^*) = 0 \ .$$

Substituting into (34) and (35) functions for expenditures and option from (30) and (33) respectively, we get the system of simultaneous equations:

(36) $$Kg^{*\beta} + \frac{1}{r} g^* = Lg^{*\beta} + \hat{z}$$
$$\beta Kg^{*\beta-1} + \frac{1}{r} = \beta Lg^{*\beta-1} = 0$$

with solution:

(37) $$L = 0\,,\; K = -\frac{1}{\beta r} g^{*1-\beta}\,,\; g^* = \frac{\beta}{\beta - 1} r\hat{z}\,.$$

Formulas (37) show that given total costs incurred by the budget capping, hence given flow costs, $r\hat{z}$, the budget has to be capped optimally at the level of government expenditures equal to g^*. The magnitude of the capped spending in stochastic case is larger than in the pure deterministic case by the ratio $q^* = \frac{\beta}{\beta - 1}$ which is analogous to the well known Tobin's q [5]. Budget capping policy implementation at the reflecting barrier g^* is shown on Figure 1.

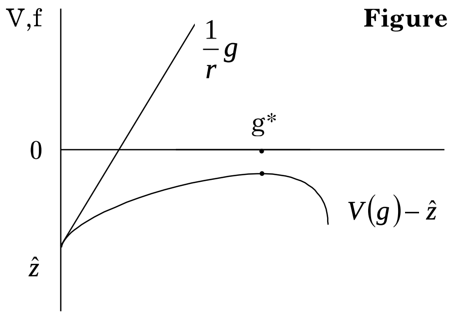

Figure 1.

Let us illustrate empirically the results just obtained. According to [12] in Russian economy ratios of federal expenditures to GDP in years 1994-1996 have variance of their logs equal to $\sigma^2 = 0{,}013$. The rate of return on real market was roughly estimated as being $r = 0{,}1$ or 10 per cent annually. Assuming the magnitude of admissible costs of budget capping to be $\hat{z} = 1{,}27$ (in terms of expected present value of government expenditures), or 12,7 percent of GDP in terms of their current value, the optimal ratio of budget capping is $g^* = 0{,}20$. In other words, it is optimal to perform the budget capping when federal expenditures approach 20 percent of GDP, considering the 12,7 percent decline in current production as admissible cost of such an operation.

In this particular case the maximal characteristic root is equal to $\beta = 2{,}74$ and Tobin's q^* is, $q^* = 1{,}57$, implying high effectiveness of the budget expenditures. At this optimal point expected present value of expenditures would be equal to *2,0* while $v(g^*) = 1{,}27$, or their capitalized value roughly two times higher than the current value of GDP. That gives the magnitude for consumption net of wealth contribution, $c^* - ra$, to be equal approximately to 87 percent of GDP.

For the phase of slightly declining production that is approximated by condition $Y(t) = Y = const$, the model seems to be realistic enough, and reveals one striking feature of the transition economy: as it appears, while running the budget the government has, in a sense, no freedom of choice but to *cap its expenditures* once they reach the threshold g^*. Hence the value of an option of *«the budget capping»* for the government, speaking formally, is equal to zero, $f(g) = 0$, as it follows from (37). This fact is in a sharp contrast with the government behavior on financial market where it appears to be able of maximizing both the expected present value of its debt and the corresponding option *«to stop its borrowings»* on the open market.

Separation of real and financial markets

Transition economy, at least of the Russian type of market transformations, possesses of several basic characteristics to be of greater importance for the model under consideration.

First, during the whole period of reforms real market demonstrates very loose dependence, if all, on the development of financial market[6]. Mutual independence of real and financial markets to a great extent underscored by the fact of low sensitivity of investment demand to the changes in real interest rate. Underdeveloped process of market capitalization of working capital and private wealth in general, together with relatively small size (in terms of financial instruments outstanding) of the financial market itself, might be considered as major factors accounted for the incoherent performance of real and financial markets in transition period. But every dark cloud has its silver lining, indeed, and actual separation of real and financial markets makes it possible for a government to take advantage of it, namely, to choose fiscal and monetary strategies to a large degree independently of each other, without endangering to provoke a conflict between them.

Second, financial market in transition economy has been developed as a highly asymmetric one. By this term we mean dominance of the segment of government debts in the financial market as a whole. For example, in the countries with the developed financial market, the capital market structure (in amounts of financial instruments outstanding) might be roughly represented by proportions 1:2:3:1:3 for corporate debts, government debts, common stocks, consumer and corporate loans, and mortgages, respectively. For transition economy, on the contrary, the appropriate structure of the financial market is represented by proportions 1:7:2 for corporate debts and loans on money and capital markets being taken together, and segments of government debts and stocks, respectively. The immediate implication of such asymmetric structure of capital market is that it permits the government (or central bank) to exercise a monopolistic power by manipulating with not only the supply side of its debts but influencing directly its demand side as well. It will be shown later that on such markets the government appears to be able to maximize the positive value of its option *«to stop borrowing»*, thus implementing the policy of debt stabilization with the appropriate costs being fully covered by private investors.

Third, in a transition economy money market (in macroeconomic sense of this term) is a highly imbalanced one due to mass and persistent arrears, or bad debts, the mere existence of which undermines market reforms in general. Emerging, incomplete and distorted, competition in the economy of transition had produced specific conditions under which the sharp deceleration of money supply appeared to be possible regardless the demand for money. Transition economy, at least of the kind existed in contemporary Russia, is characterized by the deeply disbalanced money market. Hence, the government receives possibility to control seigniorage issuance virtually in an unrestricted way. Strong and persistent disequilibrium on the money

[6] It is interesting to note that the inverse is not true: it is well known that the world financial crisis of 1987 did not by and large influence real market [13] in all the economies suffered from financial collapse, though in no way the assertion of their mutual independence is conceivable. Unilateral sensitivity, as it seems, suits the picture in this case: bad performance of real market is translated immediately into financial convulsions while the opposite is not always true.

market produced an entirely new «damping element» for Russian economy , known as arrears, though the better name for them would be «bad debts»[7]. On the one hand, arrears serve the role of a damping component «to restore» the balance between money supply and money demand. The arrears elimination, or restoring of the equilibrium between money supply and demand for money, might be viewed, in a sense, as a process of debt stabilization. Given the dominance of the segment of government debts, seigniorage has been used mainly in fiscal interests servicing the debt obligations of the government in transition period.

Weak interconnections between real and financial markets that exist in the economy of transition might be formalized with the *real and financial markets separability* condition:

$$\int_t^{\infty} \mathrm{E}_t[h(\tau)-g(\tau)]\exp[-r(\tau-t)]d\tau = 0\,. \tag{38}$$

Actually, constraint (38) reveals itself in the absence of the primary budget deficit. For economic situation in Russia where the major problem of the budget balancing stems from the debt service and not from financing its current expenditures, this assumption seems to be reasonable enough[8]. Imposition of this constraint on the consumption dynamics equation (19) helps to decompose it into two independent equations:

$$\begin{aligned} c(t) &= 1 + ra(t) - r\int_t^{\infty} \mathrm{E}_t\{g(\tau)\}\exp[-r(\tau-t)]d\tau \\ b(t) &= \int_t^{\infty} \mathrm{E}_t\{s(\tau)\}\exp[-r(\tau-t)]d\tau, \end{aligned} \tag{39}$$

for the real and financial markets respectively. It should be noted that the constraint (38) does not require budget to be balanced for the every moment of time but for the entire perspective only.

The system (39) tells us that economic transition demonstrates explicitly the existence of possibilities for the government to deal rather independently with real and financial markets. Would the strategy of debt financing through seigniorage issuance be chosen, then, in principle, the debt eventually might be fully paid off simultaneously with the real market normalization, meaning by the latter the attaining of the balanced budget. Under conditions described above, the major stimulus for the government (or the central bank) to supply new money, or seigniorage, is the necessity to meet its debt obligations (the components of money demand dependent

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[7] In the Western economic literature this important characteristic of a transition economy was noticed by *G. Calvo* in [14], who claimed it as probably the most important characteristics of a transition period. *E. Baranov, et al* estimate the share of barter in Russian industry as growing up to approximately 70 percent of industrial output [15, p. 7]

[8] Primary deficit in Russian economy was negative for 5 month in 1998 that amounted to -1,6 percent of GDP [16, p. 96].

upon income and interest rate are fixed due to conditions, $Y(t) = Y = const$, and $r = const > 0$). The latter seems to be in agreement with the reality of economic transition: lack of the nonmonetary sources of the budget deficit coverage due to depression forces the government to rely heavily on seigniorage as the only means of financing its debt service.

The first equation in (39) might be analyzed along the lines of the previous section and all the conclusions having been made above are still valid for this modified but qualitatively identical set-up. At the same time, along with the maximization of expected present value of budget expenditures the government simultaneously, and what is important, independently of real tax and spending instruments, becomes able to get started to stabilize its debt.

The problem of the government debt stabilization in a specific conditions of a transition economy could be stated in the following way. The government tries to meet its debt obligations by maximizing market value of the future stream of seigniorage: in reality it helps to reduce the burden of its service. In our model this consideration works rather implicitly though, due to constancy of the riskless rate of return. The government should pay out its debts while ceasing to borrow on the open market - the problem that sounds rather paradoxically. Stationary points of the debt equation provide no answer to this question: some of them are obviously either irrational - steady state debt would not be paid out, or unoptimal. It will be shown instead, that it is optimal for the rational government to use a strategy that permits to postpone stopping, alias to continue its borrowing on the open market up to a point that is *a priori* unknown.

A simple deterministic option to wait

The subject of the public debt management has periodically gained and lost its significance for economists and politicians. Historically, it was *Adam Smith* who had helped to establish the mood in the eighteenth century, when he prophesied that an enormous debt would destroy in the long run economies of all the great nations of Europe. Incidentally, another great economist, *David Ricardo*, shared Smith's forebodings to such an extent that he almost ruined his parliamentary career with a radical proposal for a once-and-for-all discharge of the existing British debt that had been accumulated in the wars against Napoleon [17].

It is a historic irony that *Ricardo's* recommendations though having been rejected by the British government were realized precisely a hundred and fifty years later in Romania. In the 80-ies Romania under regime of *Ceaucescu* supposedly striving to gain immediately an economic independence concentrated all its efforts on the immediate retirement of its huge foreign debt. That absolutely unjustified, though voluntarily self-imposed upon Romanian economy obligation, created enormous difficulties that led eventually to a virtual collapse of its economic and political regime. The then President of Romania, who like a famous character of *Moliere's* play spoke in prose without knowing it, implemented the Ricardo appeal, so to say, exactly, word for word. Romanian example, though being a tragic grotesque, testified empirically desirability of the postponement of debt stabilization, if possible, the proper debt service provided. In other words, immediate debt reimbursement might be either irrational, like in the case of Romania, or at least unoptimal. Seems that the great *David Ricardo* was not correct, when he had insisted on the immediate retirement of the entire debt simply because of the evident fact that the debt service of the growing debt (in other words with borrowing permitted) is cheaper than cost of service for the constant matured debt. Hence waiting in the process of the debt stabilization has a nonzero value that is easy to demonstrate in a simple deterministic case of a debt service where for simplicity mature debt and debt with borrowing have the same face value.

Let $F = b(0) = v(0)$ be a face value of the debt outstanding $b(t)$ at the moment $t = 0$, which would grow at the instantaneous rate, $\alpha > 0$, over the infinitesimally short time period dt, if the borrowing is permitted:

(40) $$\frac{dv}{v} = \alpha\ dt\ ;\ v(0) = F\ .$$

Equation (40) has a function

(41) $$v(t) = F \exp[\alpha t]$$

as its general solution that shows growing value of a debt with the constant rate of continuous borrowing.

Market or present values at time t :

of a debt without borrowing is $b(t) = F \exp[-rt]$, and

of a growing debt is $v(t) = F \exp[-(r-\alpha)t]$,

and both are discounted at the riskless rate of yield, $r > 0$. The cost of service for the fixed or matured debt per infinitesimally short period of time is equal to r, while the growing debt is serviced at the positive rate:

$$\delta = (r - \alpha) > 0\ .$$

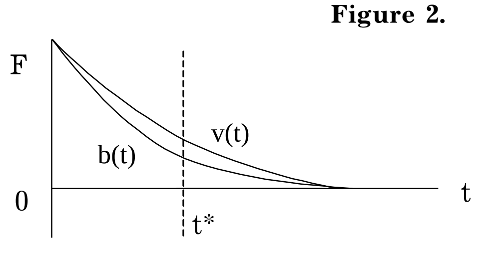
In a purely deterministic case it is possible to calculate exactly the moment of time $t*$ when it is optimal to stop to borrow by maximizing difference:

(42) $$\max f(t) = [F \exp(\alpha t) - F] \exp[-rt]\ .$$

Differentiating (42) by time at the point of optimum we get

(43) $$t* = \alpha^{-1} \ln[r/(r-\alpha)]\ .$$

Figure 2.



It follows from (43) that in general $t* > 0$, for $\frac{r}{r-\alpha} > 1$, and hence it is worthwhile to wait before to start paying out a debt totally. The immediate payoff, that is $t* = 0$, is justified only in the case of $\alpha = 0$, when the debt retirement is not accompanied with the borrowing.

Seigniorage and the debt service

In the general case of random debt and borrowing processes the government and private investors are assumed to *behave rationally* on financial market in the sense that both of them expect seigniorage to evolve in accordance with the geometric Brownian process $s(t)$. The latter, for the reasons specified above, satisfies the stochastic differential equation

$$ds = \sigma s dW \tag{44}$$

$$\mathrm{E}_t\{ds\} = 0, \; Var\{ds\} = \sigma^2 s^2 dt.$$

Equation (44) as well as similar to it (22) implies that the values of the process are known at the beginning but unknown at the end of infinitesimally short time interval. Hence the assumption of the geometric Brownian movement incorporates the government incomplete control of the seigniorage process as well as general influence of uncertainty upon it.

The second equation of the system (39) being written in the form of stochastic differential equation:

$$\frac{1}{dt}\mathrm{E}_t\{db\} = rb - s \tag{45}$$

describes the path of government debt growth and its borrowing on financial markets. It should be noted that debt service relies on seigniorage, s, as an important source of its financing. The functional

$$b(t,s) = \int_t^{\infty} \mathrm{E}_t\{s(\tau)\}\exp[-r(\tau - t)]d\tau \tag{46}$$

serves as a solution to (45) only if transversality condition

$$\lim_{\tau\to\infty} \mathrm{E}_t\{s(\tau)\}\exp[-r(\tau - t)] = 0,$$

that is similar to (16), is met .

According to (45) the government operates on an open market assuming the role of a seller of its debt obligations. At time t, in order to service a debt, rb, it has to rely on borrowing, $\frac{1}{dt}\mathrm{E}_t\{db\}$, and on printing of new money or seigniorage, s, in proportions as to satisfy the debt service requirement:

$$rb = s + \frac{1}{dt}\mathrm{E}_t\{db\}.$$

On the other hand, the long position of a government should correspond to the short position of a representative private investor operating on the market of government debts. In order to persuade her to take long position on the debt market, the government has to meet her requirement of market return rb on assets available. Thus, it has to pay her out the coupon yield, s, or seigniorage, in addition to capital gain on government bonds. In other words, looking from angles of a government or a representative private investor operating on the open market, equation (45) represents *the standard no-arbitrage condition*.

Continuation and stopping to borrow

Generally speaking, the government stops to borrow on the open market at the stationary point of (45), where the expected present value of debt ceases to change in time, or $\frac{1}{dt}\mathrm{E}_t\{db\}=0$. This requirement gives a simple relation between seigniorage and the present expected value of a debt in a steady state:

$$b(s)=\frac{1}{r}s, \tag{47}$$

where both parts of (47) are parametrized by variable t that is omitted for simplicity.

The present value of zero seigniorage is obviously equal to zero, $b(0)=0$. The point of zero seigniorage, $s=0$, might be treated as *the debt default* condition: if the government is unable to pay out its debts at $s=0$, then the market value of debt outstanding, $b(0)-F$, were to equal to $-F$, at least, theoretically. It is logical to consider the magnitude, or face value, of a bad debt accrued by the time $t=0$ to be equal to

$$\int_{-\infty}^{0}[g(\tau)-h(\tau)]\exp[r(t-\tau]d\tau=F, \tag{48}$$

which is, for given trajectories of expenditures and taxes, a constant being independent of seigniorage (no government pays interest on the money issued, at least in transition economy)[9].

It should be stressed, however, that *the total refusal of the government to issue new money transforms transition economy into a completely barter economy*[10]. The well known inefficiency of a barter, in its turn, would have imposed large sunk costs upon the transition economy due to numerous inconviniences and losses associated with it. As J.Tobin put it:« an economy with monetary institutions is different in real outcomes from a barter economy, even from an ideal frictionless barter economy…» [18]. In Russian economy costs of a barter exist in a specific form of mass and persistent arrears, *alias* bad debts, initiated incidentally by the government itself. Let us assume the magnitude of costs incurred by the absence of a seigniorage at point,

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[9] Condition of a zero primary deficit in the future that had been imposed earlier is not valid for the past, hence in general case $F>0$.

[10] The money supply shortage due to super-tight monetary policy implemented in Russia in years 1994-1998 has resulted in a barter that accounted roughly in 40 to 60 percent of the product market turnover.

$s = 0$, be constant and equal to F, in terms of their ratios to GDP. It is possible to consider these costs as alternative costs of implementation of a policy of «debt stopping», in a way, as a logical extension of a policy of «super-tight money». Such costs might serve as an important anchor for evaluating feasibility of different macroeconomic policies in a period of transition[11)].

Looking at the problem from this angle, it becomes evident that for all stationary points satisfying the inequality, $s < \hat{s} = rF$, *the immediate* debt retirement would be irrational because the market value of debt is strictly less than costs of the alternative policy implementation. Hence the policy of borrowing *should be continued*, at least up to the point where $\hat{s} = rF$. This behavior might be interpreted in another way, as follows: it is rational for the government to continue borrowing as long as lenders are willing to credit it at the going rate of interest, that is until the system would reach stationary point within the region $s < \hat{s} = rF$.

On the other hand, for all the points $s > \hat{s}$ it is impossible to say *a priori* where to stop borrowing rationally: differentiating (47) gives $\frac{db}{ds} = \frac{1}{r} > 0$, implying that larger the seigniorage, the larger would be expected present value of the debt. Hence seigniorage should be issued indefinitely in order to get maximum market value of a debt.

Meanwhile, the process of new money issuance is not an unrestricted one: pumping money into economy in the longer run lead to inflation. Rational investor expects that seigniorage would eventually approach the threshold thus triggering the burst of inflation which he would have been unable to cope with. In order to protect his additional real assets from devaluation rational investor would have stopped to buy new government debts at the same point s^*, where the following equality takes place:

$$\lim_{s \to s^*} \frac{db}{ds} = 0. \tag{49}$$

Consequently, the *degenerate stationary point* s^* that satisfies condition (49) constitutes the reflecting (the upper) barrier for the random process of seigniorage evolution. Thus issuance of seigniorage might be represented as a regulated stochastic process the values of which are confined within the lower limit (absorbing point, $s = 0$), and the upper limit, $s = s^*$, or reflecting point that should be find out.

Debt stabilization strategy

Let us now define more strictly the policy of *the debt stabilization* by which we mean *the optimal stopping of government borrowing* on the open market. While borrowing on the open market the government is assumed to retain the capability to change its policy, or speaking differently, it is assumed to hold an option *«to stop borrowing later»*. Formally, such an option might be considered as an analogue to a

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[11)] In our opinion, the economic essence of arrears in a transition economy does not permit to treat them as a modification of a «monetary without money» economy, though a controversy associated with instability of money demand in modern market economies [19] seems to be a closely related topic.

financial «call option»[12]. The government that operates in a transition economy, so to speak, «purchases» an option by paying the *«option premium»*, F, that it has actually done by creating arrears[13]. Holding such an option, in its turn, creates opportunity costs represented by seigniorage, s, that the government has to pay as the coupon yield to private investors. It follows from the previous analysis that the break-even point, $\hat{s} = rF$, might be considered as a *«strike price»*, that has to be paid in order to retire debt. When seigniorage increases to $s = s^*$, the option should be exercised, thus giving the maximal market value (expected present value of the future stream of seigniorage) of debt that covers the combined costs of abandoning an option and its premium.

In macroeconomic context, the optimal price that is paid by the government as an option holder at the exercise point, is associated with the debt stabilization by definition, meaning that the government getting stopped its practice of borrowing (and seigniorage issuance). Thus the *policy of seigniorage and debt continuation* should be compared with the *alternative policy of seigniorage and debt stopping*. In other words, the proper debt policy assessment require the comparison between costs of inflation and costs associated with arrears, alias depression and unemployment. The debt policy of a government in this aspect is represented as a seigniorage-contingent policy which is to be changed once the «seigniorage target» has been achieved.

Let us introduce nonnegative function $f(s)$ to define the value of the opportunity to borrow, or the value of an option *«to continue borrowing»*, such that $f(s) \geq b(s) - F$, and $f(0) = 0$. The nonnegativity of function $f(s)$ reflects the limited liability of the government to perform another macroeconomic policy, that is to stop borrowing. The government opportunity to borrow on the open market is, obviously a state-contingent policy: for all values of seigniorage s, $0 \leq s < s^*$, where it is not optimal to stop borrowing, the value of an option *«to continue borrowing»* is governed by the equation

$$(50) \qquad rf(s)dt = \mathrm{E}_t\{df(s)\},$$

that reflects the growing expected value of this policy implementation over the infinitesimally short period of time. On the other hand, once the «seigniorage target», $s = s^*$, is achieved, it is optimal to exercise an option, $f(s^*)$, or abandon the policy of borrowing. At this point the government gets the maximal expected present value of a future stream of seigniorage, $b(s^*)$, that makes it possible to economize on the debt service, while covering the combined costs of arrears, F, and of abandoned policy, $f(s^*)$, that is:

$$(51) \qquad b(s^*) = f(s^*) + F.$$

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[12] We treat this analogy only formally thus giving away problems of the completeness of the market, riskiness and institutional patterns of such options trading.

[13] In a transition economy the option premium that is associated with arrears, H, represents just a part of the total imbalance, $F = H + D$, where D is the face value of a debt. Since arrears do not exist in the balanced money market, $F \equiv D$.

Taking both (50) and (51) into account, the government policy of borrowing on the open market might be described by the Bellman equation:

$$f(s) = \max\{\frac{1}{r}\frac{1}{dt}\mathrm{E}_t\{df(s)\},\ b(s^*) - F\}. \quad (52)$$

Speaking the vernacular, equation (52) means that the government should continue to borrow on the open market up to the point s^*, where it becomes beneficial, by changing the gear, to get started the process of the debt stabilization. Application of the same approach to the debt dynamics process permits, due to equation (45), to construct the Bellman equation for the government debt policy:

$$b(s) = \max\{\frac{1}{r}[s + \frac{1}{dt}\mathrm{E}_t\{db(s)\}],\ f(s^*) + F\}. \quad (53)$$

Debt dynamics function

To find out optimal point for the seigniorage that supposed to be a random process satisfying (44), we have to apply again Ito's lemma to the process of dynamics for the expected present value of debt:

$$\mathrm{E}_t\{db\} = \mathrm{E}_t\{b'(s)ds + \frac{1}{2}b''(s)(ds)^2\} = \frac{1}{2}\sigma^2 s^2 b''(s)dt.$$

The dynamics of debt and seigniorage, as it was stated above, in the continuation region, $0 < s < s^*$, follows equation (45), that permits to get the particular form for the Bellman equation (53):

$$\frac{1}{2}\sigma^2 s^2 b''(s) - rb(s) + s = 0. \quad (54)$$

The solution of (54), subject to absorbing and reflecting points, is a deterministic function of a random process of seigniorage:

$$b(s) = As^{\beta} + \frac{1}{r}s, \quad (55)$$

where $\beta \equiv \beta_1 > 1$ is the (larger) real characteristic root of the equation $\frac{1}{2}\sigma^2\beta(\beta - 1) - r = 0$. The constant $A \equiv A_1$ that corresponds to this root is nonzero in general, while the constant corresponding to the second root $\beta_2 < 0$ is zero, $A_2 = 0$.

Rational behavior of a government and private investors on the debt market should be specified differently. Supposing that a government does not resell debts, its behavior might be described by a function

$$b_g(s) = \frac{1}{r}s. \quad (56)$$

In accordance to (56) the government, utilizing its monopolistic power over the price on financial assets, simply capitalizes the future flows of seigniorage in the longer run[14]. Contrary to that, a rational investor expects, and fears of, the burst of inflation capable to wipe out marginal value of her assets. In anticipation of such kind of events, she corrects (diminishes actually, as it will be shown later) the expected present value of government bonds:

$$b_p(s) = A_p s^{\beta} + \frac{1}{r}s;\ A_p < 0\,, \tag{57}$$

where negative constant A_p probability for the debt to diminish in its value somewhere in the future.

Options to continue borrowing and lending

In order to find out the debt and seigniorage stabilization point it is necessary to evaluate function $f(s)$, that was defined earlier in general form (52). While borrowing on the open market, the government holds an option of «the borrowing continuation» which has to be exercised optimally in the point s^*. In the same manner, private investors would continue to buy government bonds up to a reflecting point where they give up their option *«to continue buying government debts»*. Hence in terms of options their behavior is represented by the Bellman equation (52), implying that in the continuation region an option does not deliver any coupon yield to its holder but receives a capital gain over the infinitesimally short period:

$$rf(s)dt = \mathrm{E}_t[df].$$

According to (52) the government and private investors maximize their respective options *«to continue borrowing»* and *«to continue lending»* in respect to seigniorage. Repeating the same reasons as in previous sections, we arrive to the equation:

$$\frac{1}{2}\sigma^2 s^2 f(s) - rf(s) = 0\,, \tag{58}$$

with a solution for options in the region of continuation:

$$f(s) = Bs^{\beta}\,, \tag{59}$$

where $\beta \equiv \beta_1 > 1$ is the root of characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1) - r = 0$. In the general solution of (46) the constant $B \equiv B_1$, corresponding to the positive root, is nonzero in general, while another constant that corresponds to the negative root, $\beta_2 < 0$, should be equal to zero, $B_2 = 0$.

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[14] In the short run, as the recent actions of Russian government on financial market demonstrate, the government working through the central bank, actually, rather manipulates with the rate of return on its bonds.

Debt stabilization strategies

Let us now utilize the following considerations for defining strategies implied by the government and private investors at *the exercise point,* where the economic policy is changed. At this point the government and investors get the maximal expected present value of debts (assets for investors) that is equal to the maximal expected present value of the exercised option plus the face value of the debt. This equality forms the following value matching condition:

$$b(s^*) = f(s^*) + F . \tag{60}$$

At the optimal point the government by issuing seigniorage in amount of s^* buys out the face value of a debt and has to pay in addition the value of the abandoned opportunity to borrow. The positive difference

$$b(s^*) - F = f(s^*)$$

has to be transferred to the issuer of debts, that is to the government in this case.

The smooth pasting condition takes different forms for the government and for private investors. The government increases seigniorage being unconcerned of the future inflationary prospectives. It sells short the debts exercising its monopolistic power over their price, while considering that inflation may help it to finance debt servicing. Taking into account these considerations, we get the following smooth pasting condition for the government:

$$b'(s^*) = f'(s^*) . \tag{61}$$

For the private investor who is concerned with the marginal market value of her assets subject to inflation (49), condition (61) is modified into a different one:

$$b'(s^*) = f'(s^*) = 0 . \tag{62}$$

The government, as it was said above, does not correct its expectations of discounted value of the debt and just capitalizes the value of seigniorage. Hence we get the following system of simultaneous equations for the government:

$$\begin{aligned} &\frac{1}{r}s^* = B_g s^{*\beta} + F \\ &\frac{1}{r} = \beta B_g s^{*\beta-1}, \end{aligned} \tag{63}$$

which is satisfied for

$$B_g = \frac{1}{r\beta} s^{*1-\beta} \text{ and } s^* = \frac{\beta}{\beta-1} rF . \tag{64}$$

Note, that positive constant B_g reflects the government expectations of an increase in the option value. Position of private investors at the reflecting point,

$\tilde{s}$,that *a priori* is not ought to coincide with that for the government, is described by the system:

$$A_p\tilde{s}^{\beta}+\frac{1}{r}\tilde{s}=B_p\tilde{s}^{\beta}+F$$

(65)

$$\beta A_p\tilde{s}^{\beta-1}+\frac{1}{r}=\beta B_p\tilde{s}^{\beta-1}=0,$$

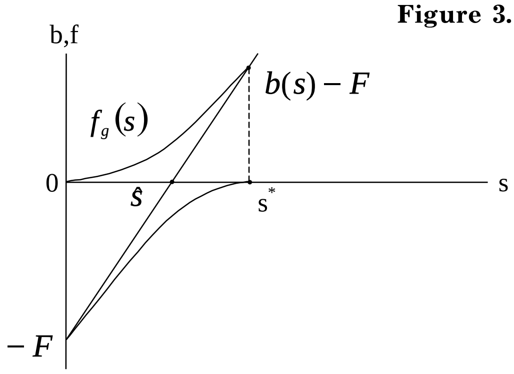
which is satisfied with the following values:

(66) $$B_p=0\,;\;A_p=-\frac{1}{r\beta}\tilde{s}^{1-\beta}\text{ and }\tilde{s}=\frac{\beta}{\beta-1}rF\,.$$

As it was noted before, rational investors expect the decrease in the value of their assets due to growing seigniorage, hence $A_p<0$.

Due to the identity of the random process of seigniorage, parameters of uncertainty and bond yield, as well as costs associated with the debt stabilization, it follows that $s^*\equiv\tilde{s}$. In other words, the government stops to borrow by capping seigniorage precisely at the same point where investors stop to lend it. Fig. 3 shows behavior of the government and private investors on financial market.

Figure 3.



It is interesting to stress, that though an economic equilibrium takes place at the optimal or exercise point, in the sense that sales of debts are equal to purchases of them, the appropriate equality does not hold for the expected present values of debt for the government and private investors, that is $b_g(s^*)\neq b_p(s^*)$ [20]. They have different priorities and, consequently, different valuations of the debt. Given costs of debt stabilization, F , it follows from (44) that

$$b_g(s^*)-f_g(s^*)=b_p(s^*)-f_p(s^*)\,,$$

and, due to $f_p(s^*)=0$, we arrive to the equality

(67) $$b_g(s^*)=b_p(s^*)+f_g(s^*)\,.$$

Equality (67) says, that at reflecting barrier (the optimal point) the combination of the monopolistic power of a government and rational behavior of private investors hedging their assets from the decrease in their real value due to probable inflation, forces private investors to compensate in full amount the government policy of stopping to borrow and stabilizing its debt. Hence targeting seigniorage at

$s = s^*$ makes it possible to wipe out arrears without producing real threat of accelerating inflation, at least due to monetary factors.

If $f(s)$ is the value of stabilization policy then who bears the cost of conducting this policy? Analysis of equations for the government and private investors at the optimal (reflecting) point

$$B_g s^{*\beta} = \frac{1}{r} s^* - F$$
$$0 = A_p s^{*\beta} + \frac{1}{r} s^* - F, \qquad (68)$$

provides the answer. At the optimal point where stabilization policy is exercised, it costs the government the expected present value of seigniorage less face value of a debt, that represented by the expression in the RHS of (68). The latter is costless for the private investors though they receive the corrected present expected value of seigniorage or RHS of the second equation in (68). The optimal point is the point of equilibrium where amount of the debts sold by the government is equal to the amount of the debt bought by the private investors that is true, if a condition $B_g = -A_p$ is satisfied. In other words, private investors are the ultimate bearers of the cost of macroeconomic stabilization.

Simple calculations based on Russian economic data help to illustrate these considerations. Real rate of yield on government bonds was equal to 60 percent annually for 1995-1997 with variance of 0,34 in logs of their price that gives $\beta = 1{,}55$ and $q^* = 2{,}8$. Hence, to liquidate arrears amounting to 30 percent of GDP it would have been necessary to get seigniorage of 50 percent to GDP approximately, while actually that ratio was several times smaller. That would suggest unambiguously in favor of the seigniorage issuance and, consequently, for mild inflation as an appropriate macroeconomic policy for Russian economy. For the same parameters we get the ratio of debt to GDP that would amount to 84 percent, which looks rather reasonable being compared to the World Bank recommended debt ratio of 40 to 60 percent for the competitive economies.

Call provisions and borrowing

It is rather evident that an opportunity to borrow has an effect on the value of a debt that is similar to the effect of call provisions. As it has been studied earlier, an opportunity to borrow has to be supported by the ability of a government to pay on its debt, and the latter could be done through seigniorage issuance that, in its turn, leads to an increase in the debt value. On the other hand, rational investors usually require the higher coupon rates on callable bonds due to the value transfer from the bond holder to the bond issuer when the debt is called out. The former is possible, again, by the issuance of seigniorage, that is through an increase in the debt value.

Let us, first, demonstrate effects of the call provisions on the value of government debt. For this purpose a somewhat simpler model is used, assuming that the debt, b, changes in accordance with the Ito equation:

(69) $$db = \sigma b dW ,$$

where $\sigma > 0$ is the standard error in the logs of debt, $W(t)$ is the standard Wiener process, and

$$\mathrm{E}[db] = 0; \ \mathrm{E}[(db)^2] = \sigma^2 b^2 dt .$$

By definition, the existence of the call provisions on government debt, $f(b)$, means that it could be called off at the face value, F, should the market value of a debt be equal to $b^* > F$. On the other hand, the debt would be bought out at market price, should the latter be lower than the face value: $b^* < F$. Hence the market value of the opportunity to call the debt optimally at b^* is subject to the following equations:

(70) $$rf(b)dt = \mathrm{E}_t[df(b)], \ b < b^* \\ f(b^*) = b^* - F, \ b = b^*,$$

which is equivalent to the Bellman equation:

(71) $$f(b) = \max\{\frac{1}{rdt}\mathrm{E}_t[df(b)], \ b^* - F\} .$$

Using the same technique as before, under requirements:

$$f(0) = 0, \ f'(b^*) = 1,$$

it is possible to estimate the optimal debt value as

$$b^* = \frac{\beta}{\beta - 1} F ,$$

and the optimal value of the call provisions as

$$f(b^*) = b^* - F = \frac{1}{\beta - 1} F ,$$

where $\beta \equiv \beta_1 > 1$ is the characteristic root of

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) - r = 0 .$$

As it is well known, the bond refunding at b^* would result in the transfer of the amount of $b^* - F = f(b^*)$ from the bond holders to the bond issuer, that is to the government. Economically speaking, the optimal value of the option to call the debt out is appeared to be the same as the value of the transfer: the difference between the market and the face value of a debt. The latter, in its turn, is justified by the increase in the debt value due to the implicit fall in the actual rate of interest. The decrease in the rate of interest could be explained, accordingly, by the increase in money supply that would have been incurred by the seigniorage issuance.

Returning to our model of seigniorage issuance (54), it is evident that it precisely the same as (71), hence implying the equivalency between borrowing and call provisions on government debt. We have to remind, that the increase in the debt value is subject to the increase in the seigniorage issuance:

$$s^* - \hat{s} = \frac{1}{\beta - 1} rF , \tag{72}$$

which in this case is equal to the cost of borrowing for the government. In order to retain its ability to pay out the debts, the government is obliged to rely on the seigniorage issuance. But the latter is appeared to be smaller than the optimal value of the opportunity to borrow, since:

$$f(s^*) = b(s^*) - F = \frac{1}{\beta - 1} F , \tag{73}$$

with «Tobin's q» equal to the reciprocal to the rate of interest:

$$q = \frac{f(s^*)}{s^* - \hat{s}} = \frac{1}{r} . \tag{74}$$

Expression (74) suggests the importance for the government of retaining the possibility to borrow on the open market (both internal and external) which is solidly based upon the confidence of lenders. Looking at the problem from this angle, it is evident that «economizing» on seigniorage is a potentially dangerous strategy because it might lead to the loss in the confidence of lenders, thus to the forceful abandonment of the policy of borrowing.

Substituting into (73) values for optimal and «refunding» seigniorage, s^* and $\hat{s}$ respectively, we get the optimal value for the opportunity to borrow, $f(b^*)$, that is equal to

$$f(b^*) = b^* - F = b(s^*) - b(\hat{s}) = \frac{1}{r} s^* - F = \frac{1}{\beta - 1} F . \tag{75}$$

It follows from (75) that the optimal value of the call provisions for the government debt is the same as the optimal value of the opportunity to borrow. That suggests the equivalence of the borrowing to the call provisions as a source of the growth of the value of the debt. The increase in the amount of seigniorage supply, in its turn, is dictated by the necessity to eliminate arrears, or bad debts, that are the main obstacle obstructing restoration of the equilibrium on the money market in the economy of transition. The value of the arrears is equal to $f(s^*)$, and their extermination would cost the government $(s^* - \hat{s})$ in terms of seigniorage. The above made analysis is illustrated on Fig. 4.

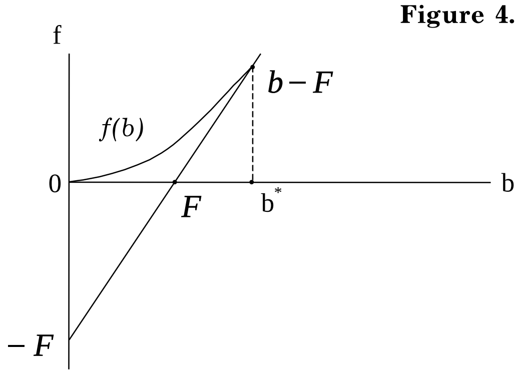


Figure 4.

Looking from this angle, the speculative operation of Russian government *cum* the Bank of Russia with government debts in May-August 1998 was theoretically deficient at least in three major points. First, it was totally wrong, from the very beginning, «to play with debts», namely, to finance the budget deficit out of borrowing on the permanent basis without a proper formation of a sinking fund, alias guaranteed debt service. Let us remind again, that borrowing makes the debt service cheaper but provide no proper answer about the debt refunding. Second, the play itself was going on the wrong side, that is in a pursue of the decrease in the value of government bonds to, say $\tilde{b} < F$, due to the increase in the market rate of interest (up to unbelievable 150 percent annually). In this case, the problem was transformed into refunding of ($F - \tilde{b}$) in the foreign loan available, as it had been intended. Taking into account very unfavorable term structure of Russian government debt, it could have been equivalent to «buying time» up to no more than half a year, in the best case. And, the third, «playing with debts on the wrong side» undermined the lenders' confidence in the government ability (and, even worse, in its desire) to pay off the debts. Hence the latter lost the mere possibility to borrow that was appeared to be the most unfortunate outcome of the entire operation.

Conclusions

The model that has been proposed in this paper describes optimal strategies for the government that operates under high degree of uncertainty on real and financial markets in a transition economy. As it appears, consumption effects of expected future fiscal policy might depend solely on future expenditures, in case of a government that would not use seigniorage as a means of payoff on its debt. The same kind of a relationship between consumption and future expenditures would be preserved under condition (38) of separability between real and financial markets. Capping of the budget expenditures seems to be independent of financial considerations, though it should be noted that the problem of coherent capping between budget and seigniorage for the model proposed was thus left unresolved.

Debt service in a transition economy might be to a large extent isolated from the fiscal policy, thus making the former a suitable instrument of fighting against arrears. Formally, the policy of an optimal debt service might be treated as a problem of optimal stopping in dynamic programming, and represented by the Bellman equation (53) or (54). The latter is, at least, instructive in a sense that it stresses the importance of seigniorage as a source of debt service, the mere fact of which is still either being ignored or underestimated by Russian government.

The policy thus prescribed might be treated as *a «seigniorage targeting policy»,* where $s*$ is a target value of a seigniorage the government has to pursue. When the target is achieved, the government receives the expected present value of a debt, $b(s^*)$, and it will cost it the sum of the policy costs, $f(s^*)$, and nominal debt, F, that is the exercise price having been paid already. It is interesting to compare this approach with the so called *«inflation targeting»* which has become the dominant theme in designing of the monetary policy in the Western literature. As it is stressed by many authors the shift towards inflation targeting has been influenced by the instability of the money demand function in the modern deregulated market economies of the West where usage of the money aggregates has become too ambiguous and misleading [21]. Inflation, $p \equiv \frac{\dot{P}}{P}$, as a target, possesses features of being a much more stable variable, as it is seen from

$$S \equiv \frac{\dot{M}}{P} = \dot{m} + pm,$$

since it is just a part of a more complex seigniorage structure that includes real money balances and their changes. On the other hand, it should be noticed that inflation is not under the direct control of the government or the central bank though seigniorage is, under the assumption of the relative independence of real and monetary markets that is valid for the economies in transition.

The seigniorage issuance as a major instrument against arrears might obviously give rise to inflation. By no means being an adherent of inflation, I should like to stress nevertheless that in the present economic context of Russian economic development, inflation would have implied by far less dangerous economic and social consequences than arrears. The policy of the arrears extermination should get the highest priority because the mere existence of mass and persistent arrears undermines market reforms in economy of transition, imposes difficulties and frustration on population, and creates dangerous social uncertainty. If, as it seems, a choice between *Scylla* of arrears and *Charybdis* of inflation should be made, then the model provides an unambiguous suggestion in support of the latter. Once being returned into the mainstream of the market reforms by restoring the balance between money supply and demand, the next phase of macroeconomic stabilization, namely curbing of inflation, should be implemented. By no means easy, appropriate measures of inflation regulation might be applied to the much more competitive economy, and arrears and the barter would not stay in the way of emerging competition.

From the macroeconomic point of view, the solution to the Bellman equation (53) signifies, in fact, the existence of appropriate boundary conditions for the optimal debt service policy implementation. It seems to be conceivable to borrow on the open market up to the point where growing seigniorage become the real menace of the burst of inflation. Theoretically, it is optimal to stop borrowing in a point where seigniorage is capped. In practice, of course, some preventive actions are to be taken in advance due to tremendous inertia in macroeconomic processes, so the actual policy would rather look like the smoothed trajectory within the explicitly defined interval between the absorbing and the «optimal stopping» points. Thus the latter could be considered as a significant anchor for designing actual fiscal and monetary policies for the economy in transition.

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