Accounting of Value Added in International Trade

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Gross exports accounting is a novel sub-area of research that seeks to allocate the value added in gross trade flows to its true country and sector of origin and country or sector of destination. Various frameworks have been recently proposed to perform such decompositions. This paper presents another effort to generalise the accounting framework so that it may be easily interpreted, customised and implemented in matrix computation software. The principal contribution is therefore a relatively simple way to derive the formulae for the decomposition of cumulative value added flows embodied in international trade. The underlying accounting approach is found to be largely similar to that of [Koopman et al., 2012; Stehrer, 2013], but the block matrix formulation allows the user to simultaneously decompose all bilateral flows at the country and/or sectoral level. The refined framework is applied to describe Russia's export performance from the global value chain perspective using the data from the World Input-Output Database (WIOD) for 2000 and 2010. According to the findings, the countries that directly receive most of Russia's exports are not exactly those that use most of Russia's value added. Russia's mining sector is found to be an intrinsic part of a complex downstream value chain where it indirectly contributes value to partner exports.

Key words: gross exports accounting; value added in trade; global value chains; inter-country input-output tables.

JEL Classification: D57, F15.

1. Introduction

For some time, simple concepts and measurements have not sufficed to describe and explain the patterns of international trade. The fragmentation of production in the late 20th and early 21st century, which some analysts consider as significant as the 19th century industrial revolution [Baldwin, 2011], led to a sustained increase in the trade of intermediate goods and

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services. It is more common nowadays that a particular product purchased for final use is the result of interactions within an inter-industry network where a multitude of producers acquire inputs from one another and add value on each subsequent production stage. Likewise, a raw material or primary input may have to virtually travel along a complex chain of industrial interactions until it is finally consumed or invested as part of a much more sophisticated product. As these interactions span across multiple borders, they are treated as global value chains or global supply chains – a term which has become representative of, if not synonymous to, the international trade [Park et al., 2013].

Case studies have shown that the traditional gross trade statistics give a misleading picture of «who produces what and for whom» (an expression from [Daudin et al., 2009]) and how the benefits from trade in the form of value added or job creation are allocated. This is perhaps most manifest in the technology-intensive industries that typically outsource many operations and rely on cross-border supplies of parts and components. In the often cited case of Apple's iPhone, the assembling and exporting economy (China) was found to directly contribute only 2% of the retail price in the destination market (U.S.), while Apple's own contribution was thought to be around 58% [Kraemer et al., 2011]. Interestingly, this is also typical for labour-intensive industries. A cost breakdown of a jacket manufactured in China and sold in the U.S. attributes only 5% to China and 86% to the U.S. [Low, 2013]. A considerable body of similar case studies based on micro data (see an overview in [Ali-Yrkko, Rouvinen, 2013]) provide useful insights but not a comprehensive solution to the issue of identifying value added in gross trade flows. For the latter, economists and statisticians now employ inter-country input-output (ICIO) tables that link flows of goods and services for intermediate and final use between industries and countries in a consistent framework originally proposed by [Leontief, 1936] and refined by [Isard, 1951; Moses, 1955; Leontief, Strout, 1963] and others. The input-output model is capable of capturing the infinite series of interactions among suppliers and consumers along the whole value chain[1].

The application of input-output techniques to international trade analyses eventually led to the emergence of a novel sub-area of research – tracing value added in trade, or gross exports accounting. The core objective is to separate net value added flows from gross trade flows as is usually done in national accounting (e.g. for GDP estimates) and to identify the origin and destination of value added in international trade. Various frameworks have been recently proposed to measure value added in trade, and the most influential contributions include [Daudin et al., 2009; Johnson, Noguera, 2012; Koopman et al., 2010, 2012; Stehrer, 2012, 2013; Wang et al., 2013]. Many of the proposed concepts have been utilised by the OECD and WTO. They jointly introduced a «Trade in Value Added» database[2] in 2013 as a stepping stone in the production of alternatively measured trade statistics.

Gross exports accounting benefits from continuous methodological advances. This paper presents another effort to generalise the accounting framework so that it may be easily interpreted, customised and implemented in matrix computation software. The principal con-

[1] [Murray, Lenzen, 2013] is a useful non-technical introduction to the multi-regional (including inter-country) input-output analysis, while the special issue of *Economic Systems Research*, 2013, vol. 25, no 1 offers a more scholarly discussion.

[2] OECD-WTO Trade in Value Added (TiVA), available at: http://stats.oecd.org/Index.aspx? DataSetCode=TIVA_OECD_WTO

tribution of this paper is therefore a relatively simple way to derive the formulae for the decomposition of cumulative value added flows embodied in international trade. The underlying accounting approach is found to be largely similar to that of [Koopman et al., 2012; Stehrer, 2013], but the block matrix formulation allows the user to simultaneously decompose all bilateral flows at the country and/or sectoral level. This leads to a discussion of the implicit and explicit sector aggregation options and an explicit derivation – for the first time, to the author's knowledge – of two matrices that account for the inter-sectoral transfer of value added in the production process of both exporting and the partner country's side.

The proposed framework is applied to quantify Russia's trade performance from the global value chain perspective. The results highlight the unobserved indirect links behind Russia's integration into the global economy. It appears that the countries that directly receive most of Russia's exports are not exactly those that use most of Russia's value added. Russia's mining sector is found to be an intrinsic part of a complex downstream value chain where it indirectly contributes value to partner exports, but the links to the final demand can also be traced.

The paper is organised as follows. Section 2 briefly overviews the frameworks proposed so far and puts this paper in the context of the ongoing research. Section 3 explains the derivation of the formulae to capture value added in international trade and a generalisation of [Koopman et al., 2012; Stehrer, 2013] framework. Section 4 applies some of the derived formulae to real world production and trade data and briefly discusses the results. Finally, Section 5 concludes.

2. Overview of the existing frameworks for the gross exports accounting

Renewed interest in ICIO analysis, partly reinforced by the release of new global IO databases[3], has been fuelling the discussion on the gross exports accounting and tracing value added in international trade. A number of frameworks have been recently proposed, and most studies refer to [Hummels et al., 1999] as the point of departure.

Hummels and his co-authors did not provide a method for the complete decomposition of gross trade flows, but proposed first measures of vertical specialisation (VS) in trade that have effectively become building blocks for the subsequent research efforts and are still widely used for global value chains analysis[4]. These measures, known as VS and VS1 can be described for any single country as follows: (1) VS accounts for the import content of a country's exports, or «how much foreign value added is required to produce a unit of direct exports?»; (2) VS1 accounts for a country's domestic value added in partner exports, or «how much domestic value added is required to produce partner countries' exports, per unit of direct exports of the country in focus?»

VS depicts a country as a recipient of foreign value added to be further processed for exports, or its relative position with respect to the upstream value chain. VS1 depicts a country as

[3] See the special issue of *Economic Systems Research*, 2013, vol. 25, no 1 for an overview.

[4] Examples include [OECD, 2013a; OECD, WTO, UNCTAD, 2013; UNCTAD, 2013]. Note that the measures of vertical specialisation appear in those publications under different names, e.g. «backward/forward participation» or «upstream/downstream component».

a supplier of domestic value added to be used in partner exports, or its relative position with respect to the downstream value chain. VS therefore relates to the backward perspective and VS1 to the forward perspective in global value chain analysis.

[Daudin et al., 2009] proposed an additional measure that is in fact a subset of VS1: domestic value added used in partner exports that ultimately returns home in final products. They call it VS1* and developed an ICIO for their computations to correct the inaccuracies in the measures derived by [Hummels et al., 1999] from single-country IO tables.

[Johnson, Noguera, 2012][5] are usually credited with the introduction of a consistent multi-country framework for the computation of the value added content of bilateral trade, or «value added exports», that describes value added produced in a source country and finally absorbed in a destination country. They proposed a ratio of value added exports to gross exports at the sector and aggregate levels called the VAX ratio as a way to address the «double-counting» problem and measure the intensity of production sharing. Johnson and Noguera generalised the computation procedures in an ICIO setting. Their contribution is also intimately related to the measurement of the factor content of trade as in [Trefler, Zhu, 2010].

[Koopman et al., 2010] developed a framework that was in many respects similar to that of Johnson and Noguera but shifted the focus of their analysis to the complete decomposition of gross exports. Their effort integrated previous literature on vertical specialisation with newer literature on value added content of trade. In brief, the core contribution of [Koopman et al., 2010] is: (1) a consistent and relatively simple method of computation of true VS and VS1 values in an inter-country setting, and (2) a decomposition that attributes all value added in a country's exports to its sources and destinations. [Koopman et al., 2010] proposed a breakdown of gross exports into three basic components: domestic value added destined for direct importing partners or third countries, domestic value added that returns home from abroad, and foreign value added. A more detailed breakdown splits these three basic components into seven more detailed components.

[Koopman et al., 2012] provided a unified framework that breaks up a country's gross exports into the sum of various components that are similar to their 2010 results. They show that the value added exports, VS, VS1, and VS1* are linear combinations of these components. Their new generalised version of the gross exports accounting equation contained nine terms. Though not explicitly observed (but recognised in [Wang et al., 2013]), the 2012 version of Koopman and his co-authors' gross exports decomposition contained a conceptual deviation from the 2010 version. The 2010 paper focused on breaking down direct gross exports into value added components, whereas the 2012 paper focused on capturing both direct and indirect value added flows normalised with respect to gross exports. This is an important distinction between the two inter-related frameworks that will become apparent in the later sections of this paper.

This is also a point where interpretation becomes critical to understand the distinctive features of the existing frameworks. [Stehrer, 2012] drew a borderline between the concepts of «trade in value added» and «value added in trade» that was rather helpful to structure readers' thoughts about the subject. In bilateral trade relations, the first concept – «trade in value added» – accounts for the value added of one country directly and indirectly contained in final consumption of another country. VAX, proposed by [Johnson, Noguera, 2012], is a good exam-

[5] First draft manuscript of Johnson and Noguera dates back to 2008.

ple of the application of this concept. The second concept – «value added in trade» – calculates the value added contained in gross trade flows between two countries. Examples include VS and VS1. The two concepts address different questions and may be used for different purposes. For trade policy which usually applies to gross trade flows, the results of «value added in trade» may be more enlightening. For global value chain analysis, «trade in value added» may be a more relevant concept. [Stehrer, 2012] also carefully studied the properties of bilateral and overall trade balances in net and gross terms.

[Stehrer, 2013] applied the framework of [Koopman et al., 2012] at the bilateral level. This allowed for a detailed account of the relationships between the two concepts from [Stehrer, 2012] and the role of third countries in bilateral value added trade. Many results in the present paper are identical to those in [Stehrer, 2013], while the derivation of the relevant formulae offers certain improvements and extends to the bilateral sectoral level.

A number of recent studies review and elaborate the frameworks mentioned so far for specific analytical purposes. [Meng et al., 2012] apply «trade in value added» and «value added in trade» concepts to measure the progress of regional economic integration through cross-border value chains. They make useful observations on the calculation of «trade in value added» at the sectoral level for their alternative version of the revealed comparative advantage indicator. [Kuroiwa, 2014] applies the framework of [Koopman et al., 2012] to derive a gross exports accounting equation for the special case of IDE-JETRO's Asian Input-Output Tables, which, unlike the global ICIO tables, contain exogenous vectors of imports from and exports to the rest of the world. He then uses the equation to assess the technological intensity of China's exports. [Kuboniwa, 2014a, 2014b, 2014c] develops a theoretical discussion on the relationship between trade balances in value added and gross terms building on many of the previously discussed concepts.

In sum, complete frameworks for gross exports accounting combine both «trade in value added» and «value added in trade» concepts. However, the exact combination varies. It is important to discern two types of such frameworks for a clearer understanding of their applications.

The first type builds on [Koopman et al., 2010] and decomposes direct exports into value added terms. A normalisation with respect to total gross exports is common and each component will be bound between 0 and 100%. [Wang et al., 2013] provide the most complete generalisation of such framework up to date with a breakdown of gross exports into sixteen components at the bilateral sectoral level.

The second type of frameworks builds on [Koopman et al., 2012] and in fact decomposes direct and indirect flows of value added, not exactly gross exports, into value added components. A normalisation with respect to gross exports is possible, but some components may well exceed 100% at the sectoral and bilateral level. A normalisation with respect to total exported value added is more natural and will bind the components between 0 and 100%. It is this type of decomposition that this paper intends to further generalise. It is suggested that this type be better described as «cumulative value added accounting» rather than «gross exports accounting» of type one.

It appears that there is no single framework that would offer all-in-one solution for all analytical purposes. Each type of decomposition has its own advantages and can be best applied in certain situations but can yield less useful results in other. A researcher or data user has therefore to carefully consider the purpose of the analysis to select proper application. Be-sides, as in many input-output based applications, meaningful economic interpretation of the proposed formulations is key to their proper use for specific analysis.

3. Refined generalised framework for cumulative value added accounting

This section introduces the notation and the «minimal matrix setup» which is thought to be one of the distinctive features of the proposed generalised accounting framework. This paper does not discuss a simplified representation of a two- or three-country world. Instead, the starting point is the general case of K countries and N sectors. The setup is thought to be convenient for both reader's understanding and the implementation in matrix computation software[6]. Moreover, there are only five core matrices and a number of common matrix operators that are required for decompositions of cross-border flows of value added. The concepts of «value added at origin» and «value added at destination» are proposed for a better understanding of implicit and explicit aggregation options. This is followed by the derivation of the «basic accounting relationship» and various algebraic manipulations to provide basic and itemised decompositions of value added in exports. The «basic accounting relationship» is also used for a surprisingly compact proof of the equality of total (i.e. aggregated across partner countries) trade balances in gross and value added terms and for a concise decomposition of the bilateral trade balances. The final part of this section discusses how the proposed framework can be customised to identify the final destination of value added at the product or sector level.

3.1. Concepts, notations and the minimal matrix setup

For the purpose of a holistic value chain analysis, the global economy is modeled by a global input-output table where each product flow is attributed to a selling industry/country and purchasing industry/country for intermediate use or purchasing country for final use. There are also primary inputs to production or value added that is also allocated to a purchasing industry/country. The matrix representation of the global ICIO table for K countries and N economic sectors appears as follows:

$$\mathbf{Z}=\begin{bmatrix}\mathbf{Z_{11}} & \mathbf{Z_{12}} & \cdots & \mathbf{Z_{1k}}\\ \mathbf{Z_{21}} & \mathbf{Z_{22}} & \cdots & \mathbf{Z_{2k}}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{Z_{k1}} & \mathbf{Z_{k2}} & \cdots & \mathbf{Z_{kk}}\end{bmatrix},\qquad \mathbf{F}=\begin{bmatrix}\mathbf{f_{11}} & \mathbf{f_{12}} & \cdots & \mathbf{f_{1k}}\\ \mathbf{f_{21}} & \mathbf{f_{22}} & \cdots & \mathbf{f_{2k}}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{f_{k1}} & \mathbf{f_{k2}} & \cdots & \mathbf{f_{kk}}\end{bmatrix},\qquad \mathbf{x}=\begin{bmatrix}\mathbf{x_1}\\ \mathbf{x_2}\\ \vdots\\ \mathbf{x_k}\end{bmatrix},$$

$$\mathbf{v}=\begin{bmatrix}\mathbf{v_1} & \mathbf{v_2} & \cdots & \mathbf{v_n}\end{bmatrix}.$$

Henceforth, indices *r* and *s* denote, respectively, the selling and purchasing countries, and indices *i* and *j* denote the selling and purchasing sectors in each country (*r* and *i* therefore correspond to rows, *s* and *j* correspond to columns). In the block matrices above, each block

[6] Section 3 in [Muradov, 2014] discusses the derivation of the formulae in much more detail.

element $\mathbf{Z}_{rs}$ is an N × N matrix of intermediate demand of the purchasing country *s* from the selling country *r*, $\mathbf{f}_{rs}$ is an N × 1 vector of final demand of *s* from *r*, $\mathbf{v}_s$ is a 1 × N vector of value added used by country *s*, and $\mathbf{x}_r$ is an N × 1 vector of total output of country *r*. So, **Z** is a global KN × KN matrix of intermediate demand, **F** is a KN × K matrix of final demand, **x** is a KN × 1 column vector of total output, and **v** is a 1 × KN row vector of value added[7]. The fundamental accounting identities imply that the supply of intermediate and final products equals total output, $\mathbf{Zi} + \mathbf{Fi} = \mathbf{x}$, and the use of intermediate and primary inputs equals total input (outlays) that must also be equal to total output, $\mathbf{iZ} + \mathbf{v} = \mathbf{x}'$, where **i** is an appropriately sized summation vector.

The generalised framework in this paper will mostly work with a «minimal setup» of only five basic matrices. One of those is **F** and four other matrices are described below.

Define a KN × KN diagonal matrix of value added coefficients:

$$\mathbf{V_c} = \begin{bmatrix} \hat{\mathbf{v}}_{\mathbf{c,1}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{v}}_{\mathbf{c,2}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \hat{\mathbf{v}}_{\mathbf{c,k}} \end{bmatrix},$$

where each block $\hat{\mathbf{v}}_{\mathbf{c,s}}$ is an N × N diagonalised vector of value added coefficients, $v_{c,j} = \frac{v_j}{x_j}$. In matrix notation, $\mathbf{V_c} = \hat{\mathbf{v}}\hat{\mathbf{x}}^{-1}$.

Two KN × K matrices are required to represent total gross exports and bilateral gross exports:

$$\mathbf{E}_{tot} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{e_k} \end{bmatrix}, \qquad \mathbf{E}_{bil} = \begin{bmatrix} \mathbf{0} & \mathbf{e}_{12} & \cdots & \mathbf{e_{1k}} \\ \mathbf{e}_{21} & \mathbf{0} & \cdots & \mathbf{e_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e_{k1}} & \mathbf{e_{k2}} & \cdots & \mathbf{0} \end{bmatrix}.$$

In $\mathbf{E}_{tot}$, each N×1 block element $\mathbf{e}_r$ is equal to the sum of the international sales for intermediate and final use over all trading partners, $\mathbf{e}_{tot,r} = \sum_{s \neq r}^{K} \mathbf{Z}_{rs}\mathbf{i}_n + \sum_{s \neq r}^{K} \mathbf{f}_{rs}$. In $\mathbf{E}_{bil}$, each N × 1 block element $\mathbf{e}_{rs}$ only accounts for the bilateral flows, $\mathbf{e}_{bil,rs} = \mathbf{Z}_{rs}\mathbf{i}_n + \mathbf{f}_{rs}$, $r \neq s$, and $\mathbf{i}_n$ is an N × 1 vector of ones for the summation across sectors.

7 Value added is usually assumed to be immobile across borders. Overcoming this limitation and accounting for the income flows, beyond trade flows, is an important part of OECD and WTO joint efforts, see [OECD, WTO, 2012].

Next, find the inter-country version of the Leontief inverse which is key to the demand-driven input-output analysis:

$$\mathbf{(I-A)^{-1}} = \begin{bmatrix} \mathbf{I-A_{11}} & \mathbf{-A_{12}} & \cdots & \mathbf{-A_{1k}} \\ \mathbf{-A_{21}} & \mathbf{I-A_{22}} & \cdots & \mathbf{-A_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{-A_{k1}} & \mathbf{-A_{k2}} & \cdots & \mathbf{I-A_{kk}} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{L_{11}} & \mathbf{L_{12}} & \cdots & \mathbf{L_{1k}} \\ \mathbf{L_{21}} & \mathbf{L_{22}} & \cdots & \mathbf{L_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L_{k1}} & \mathbf{L_{k2}} & \cdots & \mathbf{L_{kk}} \end{bmatrix} = \mathbf{L}.$$

$\mathbf{A}_{rs}$ blocks are N × N technical coefficient matrices that relate intermediate inputs to total output $a_{ij} = \frac{z_{ij}}{x_j}$ (so $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$). Leontief inverse $\mathbf{L}$ is a KN × KN multiplier matrix that allows total output to be expressed as a function of final demand: $\mathbf{x} = \mathbf{Ax} + \mathbf{Fi} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{Fi} = \mathbf{LFi}$.

This completes the setup for the derivation of the basic value added accounting formulae in this paper. It is indeed minimal and only consists of five matrices: $\mathbf{V}_c$, $\mathbf{L}$, $\mathbf{F}$, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$. One more matrix, $\hat{\mathbf{x}}$, that is the diagonalised total output, will occasionally feature the interim formulations but will eventually disappear from the final ones.

In addition to the usual matrix summation and multiplication, the framework will also require using the following operators:

- $\wedge$ – extracting a block-diagonal matrix from a block matrix or creating a block-diagonal matrix from a vector;
- $\vee$ – extracting a matrix with off-diagonal block elements from a block matrix;
- $\circ$ – block-element by block-element multiplication, known as Hadamard product in case of element-by-element multiplication;
- $'$ – transposition of a block matrix.

These four operators should be applied block-wise to the KN × KN $\mathbf{L}$ matrix and KN × K $\mathbf{F}$, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrices and $\mathbf{x}$ vector. This is an essential pre-requisite for all further manipulations with block matrices throughout this paper. The operators that handle diagonal and off-diagonal block elements, $\wedge$ and $\vee$, may apply to a single matrix, or an expression in square brackets, or both. For example, in $\overset{\vee}{\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\overset{\vee}{\mathbf{F}}\right]}$ diagonal block elements are first removed from $\mathbf{L}$ and $\mathbf{F}$. $\mathbf{V_c}$, $\overset{\vee}{\mathbf{L}}$ and $\overset{\vee}{\mathbf{F}}$ are then multiplied and, finally, diagonal block elements are removed from the result.

There is an implicit aggregation concept that defines the dimension of results and therefore needs clarifying. For the larger part of the paper (incl. subsections 3.2–3.5), the default dimension is KN × K or [country/sector] × country that may be reasonably called «value added at origin». It corresponds to the dimension of the $\mathbf{F}$, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrices and attributes each aggregated flow destined for country *s* to each sector *i* in country *r* which sources value added.

So the recipient sectors *j* on the partner countries' side are implicitly aggregated. A reverse concept is «value added at destination» which assigns aggregated value added from country *r* to product of sector *j* somehow used by country *s*. This is equal to the K × KN or country×[country/sector] dimension. In other words, the KN × K aggregation focuses on value added creation whereas K × KN focuses on value added capture. Finally, the full KN × KN or [country/sector] × [country/sector] dimension is feasible to trace both value added creation and capture. An extension to the KN × KN and K × KN dimensions is explored in subsection 3.6. Meanwhile, Fig. 1 illustrates the distinction between the two concepts which is essential for the proper interpretation of the formulations below.

Note that the application of the «value added at destination» concept requires that the matrices **F**, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ be redefined in KN × KN and the «minimal» initial setup be extended to include an additional matrix constructed from **Z**. Throughout algebraic manipulations, some matrices may therefore have to be redefined in dimensions other that the above. In such cases, for clarity, the new dimension is specified by a subscript, e.g. $\mathbf{Z}_{(KN \times K)}$ or $\mathbf{F}_{(KN \times KN)}$.

Value added at origin

Origin of value added		Destination of value added
		Country *s*: imports/use
Country *r*: exports/production, N sectors (*i*)	Sector *i*1	value added flow from *i*1
	Sector *i*2	value added flow from *i*2
	Sector *i*n	value added flow from *i*n

Value added at destination

Origin of value added	Destination of value added		
	Country *s*: imports/use, products of N sectors (*j*)		
	Product of sector *j*1	Product of sector *j*2	Product of sector *j*n
Country *r*: exports/production	value added flow in *j*1	value added flow in *j*2	value added flow in *j*n

Fig. 1. Illustration of the value added at origin and value added at destination concepts

3.2. The basic accounting relationship

Note that, by definition of gross exports:

$$\mathbf{E}_{bil} - \mathbf{E}_{tot} = \mathbf{F} + \mathbf{Z}_{(KN \times K)} - \hat{\mathbf{x}}_{(KN \times K)}, \quad (1)$$

where $\mathbf{Z}_{(KN \times K)}$ is the **Z** matrix condensed to the KN × K dimension (aggregated across the partner country *s*' sectors), and $\hat{\mathbf{x}}_{(KN \times K)}$ is a KN × K block-diagonalised vector of total output arranged in a similar way, to conform with the $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrix dimensions. The resulting matrix on both sides of (1) has bilateral trade flows in the off-diagonal block elements and total exports with the negative sign in the diagonal block elements.

Using that $\mathbf{Z}_{(KN\times K)} = \mathbf{A}\hat{\mathbf{x}}_{(KN\times K)}$, rewrite the right part of the equation as follows:

$$\mathbf{F} - \hat{\mathbf{x}}_{(KN\times K)} + \mathbf{Z}_{(KN\times K)} = \mathbf{F} - \hat{\mathbf{x}}_{(KN\times K)} + \mathbf{A}\hat{\mathbf{x}}_{(KN\times K)} = \mathbf{F} - (\mathbf{I} - \mathbf{A})\hat{\mathbf{x}}_{(KN\times K)} = \mathbf{F} - \mathbf{L}^{-1}\hat{\mathbf{x}}_{(KN\times K)}.$$

Then multiply both sides of (1), including the rewritten right side, by the value added multiplier matrix $\mathbf{V_c L}$:

$$\mathbf{V_c L}(\mathbf{E}_{bil} - \mathbf{E}_{tot}) = \mathbf{V_c L}(\mathbf{F} - \mathbf{L}^{-1}\hat{\mathbf{x}}_{(KN\times K)}).$$

A simple rearrangement gives:

$$\mathbf{V_c L E}_{bil} = \mathbf{V_c L F} + \mathbf{V_c L E}_{tot} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}. \tag{2}$$

And the «zoom-in view» on this equation is given below[8]:

$$\begin{bmatrix} \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t1} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{t1} & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{tk} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{t1} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{t1} & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{tk} \end{bmatrix} +$$

$$+ \begin{bmatrix} \mathbf{V}_{c,1}\mathbf{L}_{11}\mathbf{e}_1 & \mathbf{V}_{c,1}\mathbf{L}_{12}\mathbf{e}_2 & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{1k}\mathbf{e}_k \\ \mathbf{V}_{c,2}\mathbf{L}_{21}\mathbf{e}_1 & \mathbf{V}_{c,2}\mathbf{L}_{22}\mathbf{e}_2 & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{2k}\mathbf{e}_k \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{k1}\mathbf{e}_1 & \mathbf{V}_{c,k}\mathbf{L}_{k2}\mathbf{e}_2 & \cdots & \mathbf{V}_{c,k}\mathbf{L}_{kk}\mathbf{e}_k \end{bmatrix} - \begin{bmatrix} \mathbf{V}_{c,1}\mathbf{x}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{c,2}\mathbf{x}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_{c,k}\mathbf{x}_k \end{bmatrix}.$$

Each of the matrices above deserves a stand-alone interpretation. $\mathbf{V_c L E}_{bil}$ can be treated as a «bilateral value added in trade matrix». Each element in this matrix corresponds to both direct and indirect flows of value added that originates in sector *i* of country *r* and «lands» in country *s* to satisfy aggregate (intermediate plus final) demand in country *s*. Usually, the $\mathbf{V_c L E}_{bil}$ matrix does not feature the «value added in trade» calculations and does not explicitly appear in any of the existing frameworks. However, this matrix is useful to estimate the domestic value added embodied in direct and indirect gross trade flows. Note that the columns of $\mathbf{V_c L E}_{bil}$ (and of $\mathbf{E}_{bil}$) sum to the total imports of country *s* and the rows of $\mathbf{V_c L E}_{bil}$ sum to the domestic value added in total gross exports of country *r*, sectoral or aggregated.

[8] «Zoom-in view», or a representation of a block matrix contents, is limited to the equation (2) here. [Muradov, 2014] provides «zoom-in views» and additional descriptions of most of the matrices featuring the proposed framework.

The $\mathbf{V_cLF}$ matrix is much more familiar as it is equal to Johnson and Noguera's bilateral «value added exports» matrix used for their derivation of the VAX measure. So it can be called the «trade in value added» matrix. Each element here represents both direct and indirect flows of value added that originates in sector *i* of country *r* and «ends up» in country *s* to satisfy its final demand. The rows of $\mathbf{V_cLF}$ sum to the total value added (sectoral or aggregated) produced in country *r*. The columns of $\mathbf{V_cLF}$ sum to the total value added absorbed in country *s*.

$\mathbf{V_cLE}_{tot}$ is the «value added in total trade» matrix. [Koopman et al., 2010] proposed to use it for the computation of the multilateral VS and VS1 measures, as the column sums of the off-diagonal elements give VS and the row sums of the off-diagonal (aggregated) elements give VS1 in monetary terms. Note also that the columns of $\mathbf{V_cLE}_{tot}$ (and of $\mathbf{E}_{tot}$) sum to the total exports of country *s* and its rows sum to the total exports of country *r*'s value added (sectoral or aggregated) in gross trade flows.

$\mathbf{V_c\hat{x}}_{(KN\times K)}$ is a block-diagonal matrix of sectoral value added.

As the primary interest here is international trade, it is legitimate to consider the off-diagonal block elements only from equation (2). $\mathbf{V_c\hat{x}}_{(KN\times K)}$ then disappears from the basic accounting relationship since it only has the diagonal block elements:

$$\left[\mathbf{V_c\overset{\vee}{L}E}_{bil}\right]=\left[\mathbf{V_c\overset{\vee}{L}F}\right]+\left[\mathbf{V_c\overset{\vee}{L}E}_{tot}\right]. \tag{3}$$

The right side of equation (3) can be recognised as the sum of multilateral VAX and VS1 measures in monetary terms. This basic accounting relationship implies a straightforward interpretation: the value added that originates in sector *i* of country *r* and «lands» in country *s* via direct and indirect trade flows is equal to the value added that «ends up» in the final demand of country *s* plus the value added that is re-exported by country *s* to third countries. $\left[\mathbf{V_c\overset{\vee}{L}F}\right]$ is therefore a net term and $\left[\mathbf{V_c\overset{\vee}{L}E}_{tot}\right]=\mathbf{V_c\overset{\vee}{L}E}_{tot}$ is a double-counted term. This gives a basic decomposition of the bilateral value added in total trade into two components that can be expressed as its shares. Recall that the default dimension of the resulting matrices is KN × K.

In just three steps, equation (3) can be rearranged to express gross bilateral exports as a sum of value added components. First, note that the «bilateral value added in trade matrix» can be decomposed into two matrices that represent direct and indirect bilateral flows of value added in trade: $\mathbf{V_cLE}_{bil}=\mathbf{V_c\hat{L}E}_{bil}+\mathbf{V_c\overset{\vee}{L}E}_{bil}$. Then equation (3) may be rearranged as follows:

$$\mathbf{V_c\hat{L}E}_{bil}=\left[\mathbf{V_c\overset{\vee}{L}F}\right]+\mathbf{V_c\overset{\vee}{L}E}_{tot}-\left[\overset{\vee}{\mathbf{V_c\overset{\vee}{L}E}_{bil}}\right], \tag{4}$$

where $\mathbf{V_c\hat{L}E}_{bil}$ is the matrix of domestic value added in direct bilateral exports and $\left[\overset{\vee}{\mathbf{V_c\overset{\vee}{L}E}_{bil}}\right]$ is the matrix of domestic value added exported indirectly from sector *i* in country *r* to country *s*.

The second step involves the construction of a KN × K matrix of foreign value added in direct bilateral exports, $\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$, where $\mathbf{V}_{\mathbf{c}(N\times KN)}=[\hat{\mathbf{v}}_{\mathbf{c,1}} \quad \hat{\mathbf{v}}_{\mathbf{c,2}} \quad \cdots \quad \hat{\mathbf{v}}_{\mathbf{c,k}}]$ and $\hat{\mathbf{v}}_{\mathbf{c,s}}$ are N × N diagonalised vectors of the value added coefficients as in previous formulations. $\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$ is analogous to $\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}$, but captures all value added other than domestic in direct bilateral trade flows.

The sum of domestic and foreign value added in direct exports is equal to the total gross exports (provided that $\mathbf{v}=\mathbf{x}'-\mathbf{i}'\mathbf{Z}$), but this only holds for the aggregated gross exports. As is known, sectoral value added in exports is not equal to sectoral gross exports because of the inter-sectoral transfer of value added throughout the production process. So in the KN×K or [country/sector] × country dimension, the sum of $\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}$ and $\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$ will not yield gross sectoral exports. For example, the exports of manufactured products may embody value added created in the agriculture or services sectors. The value added originating in manufacturing tends to be less than the gross exports of manufacturing, and the value added originating in services tends to exceed the observed gross exports of services. To attain an identity with gross exports at the sectoral level, the third step requires that a correcting term be constructed to account for the difference between gross sectoral exports and sectoral value added in exports:

$$\mathbf{E}_{bil}-\left(\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}+\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}\right)=\mathbf{E}_{bil}-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\mathbf{E}_{bil}=\left(\mathbf{I}-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}.$$

This is the matrix of the inter-sectoral transfer of value added on the exporting countries' side. As is known from the existing input-output accounts, sectoral value added is usually less than total inputs minus intermediate purchases because of international trade and transport margins, import duties and statistical discrepancies. In matrix terms, the equality $\mathbf{i}'\mathbf{V_c}\mathbf{L}=\mathbf{i}'$ or $\mathbf{v}=\mathbf{x}'-\mathbf{i}'\mathbf{Z}$ does not hold. For exhaustive accuracy, $\left(\mathbf{I}-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}$ may be replaced with $\left(\left[\mathbf{i}'\hat{\mathbf{V}}_{\mathbf{c}}\mathbf{L}\right]-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}$. See [Muradov, 2014] for an extended discussion of the computation and interpretation of this matrix.

Finally, add the matrix of foreign value added in direct bilateral exports and the correcting term to both sides of equation (4) to obtain the equation for gross sectoral exports:

$$(5)\qquad \mathbf{E}_{bil}=\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]+\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]+\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}+\left(\mathbf{I}-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}.$$

An aggregation to the K × K or country×country dimension will drop the correcting term. Construct a sector-wise aggregation matrix $\mathbf{S}_n$:

$$\mathbf{S}_n = \begin{bmatrix} \mathbf{i} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{i} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{i} \end{bmatrix},$$

where $\mathbf{i}$ are N × 1 summation vectors. Aggregate with respect to the exporting sectors. Since the columns of the value added multiplier matrix $\mathbf{V_c L}$ theoretically add up to one, the last term in (5) becomes zero:

$$\text{(6)} \qquad \mathbf{S}'_n\mathbf{E}_{bil} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{S}'_n\overset{\vee}{\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]} + \mathbf{S}'_n\overset{\wedge}{\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]}\mathbf{E}_{bil},$$

which translates the basic relationship (3) into a decomposition of bilateral gross exports at the country level, in a way largely similar to [Koopman et al., 2012; Stehrer, 2013]. The components can now be expressed as ratios (rather than shares) to gross exports and some of these ratios may well exceed 1. This formulation also links various measures known from the literature on gross exports accounting or vertical specialisation in their monetary form: the first term on the right side is the bilateral VAX measure, the second is VS1, and the fourth is VS. The third term may be treated as a «reversed VS1» because it represents the exporter's value added that flows from third countries to partners, i.e. in a direction that is opposite to VS1. So these four measures from (6) add up to the aggregate gross exports:

GROSS (BILATERAL) EXPORTS = VAX + VS1 – «reversed VS1» + VS

3.3. The itemised accounting of the bilateral value added flows

It is possible to split the $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]$ and $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]$ terms from the basic accounting relationship (3) into various components. The former can be expressed as follows:

$$\text{(7)} \qquad \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] = \overset{\vee}{\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right]} + \overset{\vee}{\left[\mathbf{V_c}\mathbf{L}\hat{\mathbf{F}}\right]} + \overset{\vee}{\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}} - \mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}\right]} = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \overset{\vee}{\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}\right]}.$$

The resulting terms on the right side need careful interpretation. The first term $\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}$ captures the value added that originates in sector *i* of country *r* and is embodied in products made in country *r* for final demand in country *s*. The second term $\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}$ captures the value added that originates in sector *i* of country *r* and is embodied in products made in country *s* for final demand in country *s*. The third term $\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]$ captures the value added that originates in sector *i* of country *r* and is embodied in products made in third countries for final demand in country *s*. The principal distinction between these terms is therefore in the place where intermediate products are transformed into final products: in the exporting country *r*, partner country *s* or third countries. Note that only the first term $\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}$ is in fact an export flow of final products while two other terms represent exports of intermediates that are finally absorbed in the partner country.

Another manipulation will discern two components in $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]=\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}$:

$$\text{(8)}\qquad \mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}=\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}+\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right].$$

The first term, $\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}$, is a matrix of value added that originates in sector *i* of the exporting country *r* and returns home via gross exports from the partner country *s* («reflected value added»). Note, again, that $\circ$ and ′ signify, respectively, block-by-block multiplication and block-by block transposition. Within block elements, normal matrix multiplication rules hold. The second term in (8), $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$, is a matrix of value added that originates in sector *i* of the exporting country *r* and is re-exported by partner country *s* to third countries («redirected value added»).

Finally, compile a new equation for a more profound decomposition of bilateral domestic value added in trade flows from (3), (7) and (8):

$$\text{(9)}\qquad \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]=\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}+\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}+\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]+\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}+\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right].$$

The corresponding equation for gross exports is as follows:

$$\text{(10)}\qquad \begin{aligned}\mathbf{E}_{bil}&=\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}+\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}+\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]+\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}+\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]-\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]+\\&+\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}+\left(\mathbf{I}-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}.\end{aligned}$$

The last term in (10), again, accounts for the inter-sectoral transfer of value added from the sector of origin throughout the production process on the exporting country's side. The sector-wise aggregation to the K × K or country × country dimension will remove this term:

$$\mathbf{S}'_n\mathbf{E}_{bil} = \mathbf{S}'_n\mathbf{V}_\mathbf{c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{S}'_n\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{S}'_n\check{\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\check{\mathbf{F}}\right]} + \mathbf{S}'_n\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} + \mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_\mathbf{c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] - \tag{11}$$

$$- \mathbf{S}'_n\check{\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]} + \mathbf{S}'_n\left[\widehat{\mathbf{V}_{\mathbf{c}(N\times KN)}}\check{\mathbf{L}}\right]\mathbf{E}_{bil}.$$

Equation (11) yields the results that are identical to those in [Stehrer, 2013] (see equation 9), and the only difference is that in the latter study, double counted terms are split into the final and intermediate components, using that $\mathbf{E}_{bil} = \check{\mathbf{F}} + \check{\mathbf{Z}}_{(KN\times K)} = \check{\mathbf{F}} + \check{\left[\mathbf{A}\hat{\mathbf{x}}_{(KN\times K)}\right]}$.

The above derivation of the gross exports accounting equation reveals that it is in fact a result of the decomposition of cumulative value added not direct exports flows. That's the reason why bilateral trade between the partner country and third countries – captured by $\check{\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\check{\mathbf{F}}\right]}$, $\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_\mathbf{c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$ and $\check{\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]}$ – appears in this formula for bilateral gross exports which may first seem counter-intuitive. This is also the reason why the range of individual components in (10) and (11) expressed as the ratios to gross exports is not confined to 0–100%, so a normalisation with respect to gross exports will give ratios rather than shares. As noted in section 2, for a decomposition of direct bilateral exports into detailed value added components that are bound between 0 and 100% one should use the framework developed in [Wang et al., 2013].

Lastly, note that the difference between $\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_\mathbf{c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$ and $\check{\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]}$ gives the balance of trade in the exporting country's value added between the partner country and third countries. The next subsection will reveal that the difference between these terms equals zero after aggregating across partner countries.

3.4. Aggregation across partner countries: the decomposition of value added in total exports[9]

As noted earlier, the row sums of $\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}$, or the aggregation across partner countries, equal domestic value added in total gross exports of country r, inclusive of reflected value added given by the diagonal block elements. Removing the reflected value added results in $\check{\left[\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}\right]}$. At the sectoral level, the aggregation of this matrix leads to a decomposition of value added that originates in sector i of country r and is sent to all partner countries via direct and

[9] The proposed framework also allows for an aggregation across exporting countries, i.e. a decomposition of value added in total imports. As this is less relevant to the scope of this paper, the decomposition of value added in total imports is not discussed here, but the interested reader may refer to the subsection 3.5 in [Muradov, 2014].

indirect exports. Below, the aggregation results are shown for both the basic accounting equation (3) and the itemised version (9):

$$\text{(12)} \qquad [\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{bil}]\mathbf{i} = [\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{F}]\mathbf{i} + [\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot}]\mathbf{i},$$

$$\text{(13)} \quad [\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{bil}]\mathbf{i} = \mathbf{V}_{\mathbf{c}}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \left[\check{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\check{\mathbf{F}}}\right]\mathbf{i} + \left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} + \left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i},$$

where **i** is a K × 1 summation vector. The interpretation of the individual terms is similar to (3) and (9), with respect to all trading partners. Pre-multiplication by the sector-wise aggregation matrix **S**′ will yield a country level decomposition.

The same type of aggregation applies to equations (5) and (10) to express total sectoral gross exports as the sum of the value added components. However, the aggregation at the country level, based on equations (6) and (11), is of particular interest. The condensed form is:

$$\text{(14)} \qquad \mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i} = \mathbf{S}'_n[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{F}]\mathbf{i} + \mathbf{S}'_n\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i} - \mathbf{S}'_n\left[\check{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{bil}}\right]\mathbf{i} + \mathbf{S}'_n\left[\hat{\mathbf{V}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i}.$$

And the itemised form is given by:

$$\text{(15)} \qquad \begin{aligned} \mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i} &= \mathbf{S}'_n\mathbf{V}_{\mathbf{c}}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\left[\check{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\check{\mathbf{F}}}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} + \\ &+ \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} - \mathbf{S}'_n\left[\check{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{bil}}\right]\mathbf{i} + \mathbf{S}'_n\left[\hat{\mathbf{V}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i}. \end{aligned}$$

The fifth term in the above equation $\mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$ is country *r*'s value added re-exported by partner country *s* to third countries. The sixth term $\mathbf{S}'_n\left[\check{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{bil}}\right]\mathbf{i}$ is country *r*'s value added indirectly exported via third countries to partner country *s*. At the aggregate country level, these terms equal each other, hence the balance of trade in exporting country *r*'s value added among all partners is zero (for an explicit proof, see [Muradov, 2014]). Equation (14) may also be rewritten in terms of the measures known from the literature on trade in value added and vertical specialisation:

$$\text{GROSS (TOTAL) EXPORTS} = \text{VAX} + \text{VS1}^* + \text{VS}.$$

3.5. A note on total and bilateral trade balances

Recall that the columns of $\mathbf{V_c L E}_{tot}$ sum to the total exports of country *s* (and also value added in total exports, provided again that $\mathbf{i}'\mathbf{V_c L} = \mathbf{i}'$ or $\mathbf{v} = \mathbf{x}' - \mathbf{i}'\mathbf{Z}$) whereas the columns of $\mathbf{V_c L E}_{bil}$ sum to the total imports (also value added in total imports) of *s*. Also note that the diagonal block elements which are the only elements in the columns of $\mathbf{V_c \hat{x}}_{(KN\times K)}$ can be interpreted as the sum of the value added generated in country *s* by both domestic and foreign final demand (which is equal to the sum across the rows of $\mathbf{V_c LF}$). And the columns of $\mathbf{V_c LF}$ sum to the total value added absorbed in country *s*. Then the basic equation (2) can be simply rearranged in the 1×K dimension to show:

$$\mathbf{i}'\mathbf{V_c L E}_{tot} - \mathbf{i}'\mathbf{V_c L E}_{bil} = \mathbf{i}'\mathbf{V_c \hat{x}}_{(KN\times K)} - \mathbf{i}'\mathbf{V_c LF},$$

or

$$\mathbf{i}'\mathbf{E}_{tot} - \mathbf{i}'\mathbf{E}_{bil} = \mathbf{i}'\mathbf{V_c \hat{x}}_{(KN\times K)} - \mathbf{i}'\mathbf{V_c LF}. \tag{16}$$

The left side of the equation (16) is a difference between total gross exports and total gross imports of country *s*, or a 1 × K vector of trade balances in gross terms. Likewise, the right side gives the difference between total value added generated and total value added absorbed in country s, i.e. a 1 × K vector of trade balances in value added terms. This completes the proof of the equality of trade balances in gross and net terms and succinctly confirms the earlier results of [Stehrer, 2012, 2013; Kuboniwa, 2014b, 2014c].

Bilateral trade balances in the K × K matrix form can be calculated as differences between relevant bilateral matrices and their transposes. For gross trade balances this can be expressed as:

$$\mathbf{S}_n'\mathbf{E}_{bil} - \left(\mathbf{S}_n'\mathbf{E}_{bil}\right)' = \mathbf{S}_n'\mathbf{E}_{bil} - \mathbf{E}_{bil}'\mathbf{S}_n. \tag{17}$$

Using (2) and (6) gross trade balance can be decomposed into various components:

$$\mathbf{S}_n'\mathbf{E}_{bil} - \left(\mathbf{S}_n'\mathbf{E}_{bil}\right)' = \left[\mathbf{S}_n'\mathbf{V_c LF} - \left(\mathbf{S}_n'\mathbf{V_c LF}\right)'\right] + \left[\mathbf{S}_n'\mathbf{V_c L E}_{tot} - \left(\mathbf{S}_n'\mathbf{V_c L E}_{tot}\right)'\right] -$$

$$- \left[\mathbf{S}_n'\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil} - \left(\mathbf{S}_n'\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right)'\right] + \left[\mathbf{S}_n'\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil} - \left(\mathbf{S}_n'\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}\right)'\right].$$

The matrix of bilateral balances of value added in gross trade can be computed as:

$$\mathbf{S}_n'\mathbf{V_c L E}_{bil} - \left(\mathbf{S}_n'\mathbf{V_c L E}_{bil}\right)' = \mathbf{S}_n'\mathbf{V_c L E}_{bil} - \mathbf{E}_{bil}'\mathbf{L}'\mathbf{V_c S}_n \tag{18}$$

or in the decomposed form,

$$\mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil} - \left(\mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}\right)' = \left[\mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{F} - \left(\mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{F}\right)'\right] + \left[\mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{tot} - \left(\mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}\right)'\right].$$

Finally, the matrix of bilateral balances of trade in value added is:

$$\mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{F} - \left(\mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{F}\right)' = \mathbf{S}_n'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{F} - \mathbf{F}'\mathbf{L}'\mathbf{V}_\mathbf{c}\mathbf{S}_n. \tag{19}$$

It is therefore evident that these three types of bilateral trade balances for a pair of countries are not equal unless under very special conditions. For example, the gross trade balance (22) equals the balance of trade in value added (24) only if the sum of respective elements in the matrices of bilateral balances of domestic re-exported value added, domestic re-directed value added and foreign value added is zero.

3.6. Extension to the KN × KN and K × KN dimensions and «value added at destination»

The discussion has so far focused on the decomposition of value added flows that originate in sector *i* of exporting country *r* and «end up» or «land» in partner country *s*. It therefore attributed all value added component flows to their country/sector of origin. An extension to the KN × KN dimension and aggregation to the K × KN dimension would capture all value added created in country *r* embodied in products of sector *j* consumed or re-exported by partner country *s*. Such change of perspective is not a trivial exercise and requires an extension to the «minimal» initial setup.

Decompositions of value added at destination rather than at origin have been suggested on an *ad hoc* basis in the literature on trade in value added. [Koopman et al., 2010] propose to aggregate across exporting country sectors and disaggregate the partner country sectors in a matrix similar to the «value added in total trade» $\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}$ matrix in this paper. They treat it as a «sectoral measure of value-added trade in global value chains» (see formula 12 in [Koopman et al., 2010] for the two-country case). [Meng et al., 2012] briefly discuss similar type of disaggregation applied to their sectoral «trade in value added» measure that they use to derive alternative, TiVA-based version of revealed comparative advantage indicators (equations 12–13 in [Meng et al., 2012]).

This paper makes a systematic effort to sort out the implicit aggregation in previously used measures in a block matrix environment. First, convert **F**, $\hat{\mathbf{x}}_{(KN\times K)}$, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrices from the KN × K to the KN × KN dimension:

$$\mathbf{F}_{(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{f}}_{11} & \hat{\mathbf{f}}_{12} & \cdots & \hat{\mathbf{f}}_{1k} \\ \hat{\mathbf{f}}_{21} & \hat{\mathbf{f}}_{22} & \cdots & \hat{\mathbf{f}}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{f}}_{k1} & \hat{\mathbf{f}}_{k2} & \cdots & \hat{\mathbf{f}}_{kk} \end{bmatrix}, \qquad \hat{\mathbf{x}}_{(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{x}}_1 & 0 & \cdots & 0 \\ 0 & \hat{\mathbf{x}}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\mathbf{x}}_k \end{bmatrix},$$

$$\mathbf{E}_{tot(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{e}}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{e}}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \hat{\mathbf{e}}_\mathbf{k} \end{bmatrix}, \quad \mathbf{E}_{bil(KN\times KN)} = \begin{bmatrix} \mathbf{0} & \hat{\mathbf{e}}_{12} & \cdots & \hat{\mathbf{e}}_{1\mathbf{k}} \\ \hat{\mathbf{e}}_{21} & \mathbf{0} & \cdots & \hat{\mathbf{e}}_{2\mathbf{k}} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{e}}_{\mathbf{k}1} & \hat{\mathbf{e}}_{\mathbf{k}2} & \cdots & \mathbf{0} \end{bmatrix}.$$

The above conversion of **F**, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ and **x** is for computational purpose only, to keep the sectoral dimension of results, and does not involve a meaningful interpretation.

Now **Z**, $\mathbf{F}_{(KN\times KN)}$, $\mathbf{E}_{tot(KN\times KN)}$, $\mathbf{E}_{bil(KN\times KN)}$ and $\hat{\mathbf{x}}_{(KN\times KN)}$ are all KN × KN matrices. Owing to the above specification, all blocks in $\mathbf{F}_{(KN\times KN)}$, $\mathbf{E}_{tot(KN\times KN)}$, $\mathbf{E}_{bil(KN\times KN)}$ and $\hat{\mathbf{x}}_{(KN\times KN)}$ contain either diagonal elements only or zeros except **Z** where blocks contain nonnegative values in all or many of the elements. For the equation (1) to hold in KN × KN dimension, one more term is required to offset the presence of the off-diagonal elements in each block of **Z**:

$$\mathbf{Z}^* = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1\mathbf{k}} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2\mathbf{k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{\mathbf{k}1} & \mathbf{Z}_{\mathbf{k}2} & \cdots & \mathbf{Z}_{\mathbf{kk}} \end{bmatrix} - \begin{bmatrix} \widehat{[\mathbf{Z}_{11}\mathbf{i}]} & \widehat{[\mathbf{Z}_{12}\mathbf{i}]} & \cdots & \widehat{[\mathbf{Z}_{1\mathbf{k}}\mathbf{i}]} \\ \widehat{[\mathbf{Z}_{21}\mathbf{i}]} & \widehat{[\mathbf{Z}_{22}\mathbf{i}]} & \cdots & \widehat{[\mathbf{Z}_{2\mathbf{k}}\mathbf{i}]} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{[\mathbf{Z}_{\mathbf{k}1}\mathbf{i}]} & \widehat{[\mathbf{Z}_{\mathbf{k}2}\mathbf{i}]} & \cdots & \widehat{[\mathbf{Z}_{\mathbf{kk}}\mathbf{i}]} \end{bmatrix}.$$

Then the equation (1) in KN × KN dimension is as follows:

(20) $$\mathbf{E}_{bil(KN\times KN)} - \mathbf{E}_{tot(KN\times KN)} = \mathbf{F}_{(KN\times KN)} + \mathbf{Z} - \hat{\mathbf{x}}_{(KN\times KN)} - \mathbf{Z}^*.$$

The same manipulation applies as in subsection 3.2, and the difference is that one more term appears on the right side:

(21) $$\mathbf{V_c L E}_{bil(KN\times KN)} = \mathbf{V_c L F}_{(KN\times KN)} + \mathbf{V_c L E}_{tot(KN\times KN)} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN\times KN)} - \mathbf{V_c L Z}^*.$$

The last term accounts for the inter-sectoral transfer of value added embodied in intermediate products on their way from the sector of origin i to the sector of destination j. So, this is the inter-sectoral transfer of value added on the partner countries' side.

Finally, removing the diagonal block elements yields the basic accounting relationship in the KN × KN or full [country/sector] × [country/sector] dimension:

(22) $$\overset{\vee}{\left[\mathbf{V_c L E}_{bil(KN\times KN)}\right]} = \overset{\vee}{\left[\mathbf{V_c L F}_{(KN\times KN)}\right]} + \overset{\vee}{\left[\mathbf{V_c L E}_{tot(KN\times KN)}\right]} - \overset{\vee}{\left[\mathbf{V_c L Z}^*\right]}.$$

All subsequent equations for the value added or gross exports accounting as in subsections 3.2–3.4 in the KN × KN dimension should include the last term from (22). In the first two matrices on the right side of the equation (22), each element should be interpreted as the value added originating in sector *i* of country *r* embodied in product of sector *j* used by country *s* for domestic consumption or re-exports. The last term accounts for the re-allocation of value added originating in sector *i* of country *r* resulting from the inter-sectoral flows of intermediates for which country *s* is responsible. Appendix D in [Muradov, 2014] explores the properties of this matrix in more detail.

Pre-multiplication of (22) and any derivative equation by the sector-wise aggregation matrix $\mathbf{S}'_n$ will condense the results to the K × KN or country × [country/sector] dimension:

$$\mathbf{S}'_n\left[\mathbf{V_c L}\check{\mathbf{E}}_{bil(KN\times KN)}\right]=\mathbf{S}'_n\left[\mathbf{V_c L}\check{\mathbf{F}}_{(KN\times KN)}\right]+\mathbf{S}'_n\left[\mathbf{V_c L}\check{\mathbf{E}}_{tot(KN\times KN)}\right]-\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right], \quad (23)$$

where each element in any resulting matrix should be interpreted as the value added originating from (all sectors of) country *r* embodied in product of sector *j* used by country *s* for domestic consumption or re-exports. Again, the re-allocation term accounts for the inter-sectoral value added flows on the partner countries' side. Note that the aggregation of (22) and (23) across recipient country's sectors, respectively, to the KN × K and K × K dimensions, i.e. post multiplication by $\mathbf{S}_n$, will make the last term equal to zero.

One should also note that a transformation of (22) into a gross exports accounting equation in the KN × KN dimension will have no meaningful interpretation because of the presence of zeroes in the off-diagonal elements of each block in $\mathbf{E}_{bil(KN\times KN)}$. However, an aggregated version in K × KN dimension can be interpreted in terms of the total value added components embodied in products received at the partner side.

This completes the discussion of the generalised framework proposed in this paper for the accounting of value added in international trade.

4. Application and selected results: Russia in global value chains

For an application of the proposed value added accounting framework, this section focuses on Russia's global and bilateral exports. Previous studies have revealed an extremely high content of domestic value added in Russia's gross exports. As an upstream natural resource producer, Russia was found to have a large share of its domestic value added absorbed by direct importers, delivered via intermediate rather than final products. A significant portion of Russia's intermediate exports were also used by other countries to produce their intermediate goods exports [Koopman et al., 2010, 2012; OECD 2013b].

In conventional analysis of international trade, exports and exporters of energy and other natural resources are often isolated or neglected. However, a brief discussion below shows that the natural resource exporters like Russia may be an interesting case for the analysis of global value chains. In particular, Russia's case will show how the value added generated in a resource-extracting sector is circulated through partner and third countries' trade, or downstream value chain.

The formulae derived through the previous section are tested here with the data from the World Input-Output Database (WIOD). The WIOD database contains a series of national and inter-country supply, use and input-output tables supplemented by sets of socio-economic and environmental indicators for the period from 1995 to 2011. The data covers 27 European

Union member states, 13 other major non-European economies plus estimates for the rest of the world and discerns 35 industries based on NACE revision 1 which corresponds to ISIC revision 3[10].

The results are estimates that necessarily reflect the assumptions that the WIOD compilers had to make in the process of creating and balancing their ICIO tables, including Russia's input-output tables[11].

4.1. Russia's global and bilateral exports: does the accounting approach matter?

As follows from the previous section, three concepts may be applied to measure total and bilateral trade flows:

- «gross exports» – that is usual gross trade statistics,
- «value added in exports» – that is gross trade flows reallocated to the countries of origin of value added contained therein, and
- «exports of value added» – that is gross trade flows reallocated to the countries of origin of value added with the double-counted flows removed.

This involves computing three vectors, $\mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i}$, $\mathbf{S}'_n[\mathbf{V}_c\check{\mathbf{L}}\mathbf{E}_{bil}]\mathbf{i}$ and $\mathbf{S}'_n[\mathbf{V}_c\check{\mathbf{L}}\mathbf{F}]\mathbf{i}$, and – for cross-country comparison – the shares of each element with respect to the column sums. Note that the reflected exports (diagonal elements in the respective matrices) are not accounted for, to conform with the basic accounting relationship (3). Accounting for the reflected exports may be important for such country as the United States, but does not affect the order of the results.

Using these three concepts and WIOD data for 2010, Table 1 ranks world top 20 exporters. Change of concept affects the relative importance of most countries in the global trade, though the differences are not dramatic. China overtook the United States as the largest exporter in terms of gross exports (10,9% vs. 10,2% of global exports), but lacked behind the United States as the country of origin of value added embodied in those exports (11,1% vs. 11,9%). Finally, China and the United States performed almost equally well in generating net value added for consumption abroad (11,7% of global traded value added, but China actually led with a negligible 0,02% advantage).

In Table 1, moving from the gross exports to domestic value added in exports measurement means removing foreign value added in national exports and adding domestic value added in partner exports (less those that return home). In the language of the literature on vertical specialisation, this means subtracting VS and adding VS1 (again, corrected for reflected exports). So net (indirect) exporters of value added – those who indirectly supply more domestic value added than directly receive foreign value added – usually raise to higher ranks. In 2010, Russia is the 11th global exporter in gross terms and 6th in terms of domestic value added in exports with the total exports indicator raised by 42%, or from 2,3 to 3,6% of the global ex-

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[10] For more information on the WIOD project, see [Timmer, 2012; Dietzenbacher et al., 2013] and the WIOD website www.wiod.org.

[11] The only supply and use table officially available to the WIOD project for Russia was dated 1995. For a discussion of the issues related to the use of this table for the modelling of a consistent time series of Russia's supply and use tables, see [Baranov et al., 2014].

ports. Similarly, Australia climbs up from the 17th to 12th position and its contribution to the global exports raises from 1,7 to 2,2%. The opposite examples are most notably given by Belgium and Chinese Taipei.

Table 1.

Twenty largest global exporters in 2010 and three measurement options (million US$ and percentage of total world exports, current prices)

Gross exports				Domestic value added in all exports				Exports of domestic value added			
No.	Exporter	US$ mln	%	No.	Exporter	US$ mln	%	No.	Exporter	US$ mln	%
1	China	1743486	10,9	1	United States	1767249	11,9	1	China	1300561	11,7
2	United States	1634458	10,2	2	China	1645795	11,1	2	United States	1298542	11,7
3	Germany	1391739	8,7	3	Germany	1195077	8,0	3	Germany	903105	8,1
4	Japan	835356	5,2	4	Japan	918590	6,2	4	Japan	690182	6,2
5	United Kingdom	617535	3,9	5	United Kingdom	620217	4,2	5	United Kingdom	452148	4,1
6	France	609074	3,8	6	Russia	528076	3,6	6	France	394262	3,6
7	Korea	519545	3,2	7	France	513184	3,5	7	Italy	346051	3,1
8	Italy	514168	3,2	8	Italy	444477	3,0	8	Canada	337358	3,0
9	Netherlands	468328	2,9	9	Canada	422439	2,8	9	Russia	330619	3,0
10	Canada	449279	2,8	10	Korea	386228	2,6	10	Korea	287267	2,6
11	Russia	371743	2,3	11	Netherlands	352855	2,4	11	Netherlands	258148	2,3
12	Belgium	322585	2,0	12	Australia	323310	2,2	12	Australia	229948	2,1
13	Spain	322167	2,0	13	Spain	276740	1,9	13	India	220449	2,0
14	Chinese Taipei	311633	1,9	14	India	275697	1,9	14	Spain	212852	1,9
15	India	308576	1,9	15	Brazil	248603	1,7	15	Mexico	185057	1,7
16	Mexico	286285	1,8	16	Mexico	231554	1,6	16	Brazil	183369	1,7
17	Australia	273733	1,7	17	Chinese Taipei	222378	1,5	17	Belgium	161346	1,5
18	Brazil	232982	1,5	18	Belgium	221631	1,5	18	Chinese Taipei	155067	1,4
19	Sweden	212123	1,3	19	Indonesia	199799	1,3	19	Indonesia	145827	1,3
20	Ireland	197741	1,2	20	Sweden	177034	1,2	20	Sweden	130802	1,2

Note: the Rest of the World is dropped from the list of exporters.
Source: WIOD database, author's calculations.

Moving further right in the table, from domestic value added in exports to exports of domestic value added means removing the domestic value added that circulates through the downstream value chain via intermediate products and is therefore double counted. Then net exporters of value added would typically step back or, at best, retain their ranks. In 2010, Russia being the 6th largest country in terms of domestic value added in exports (contribution 3,6%) becomes the 9th exporter of net value added (contribution 3,0%). Australia remains to be the 12th largest exporter with a slightly lower contribution in terms of net value added (2,2% turns to 2,1%).

In Table 2, same measurement concepts are applied to Russia's bilateral exports. The three approaches to identify the largest export partners yield three different results: the principal export market in gross terms is Italy (8,8%), the principal destination for Russia's value added embodied in gross exports is China (7,0%), and the most important final destination for Russia's value added is the United States (9,6%). The differences as we will see in more detail below stem from the relative position of Russia's trading partners as absorbers or re-exporters of value added.

Table 2.

Russia's ten largest trade (export) partners in gross and value added terms, 2010 (million US$ and percentage of Russia's total exports, current prices)

Gross exports				Domestic value added in all exports				Exports of domestic value added			
No.	Partner	US$ mln	%	No.	Partner	US$ mln	%	No.	Partner	US$ mln	%
1	Italy	32783	8,8	1	China	37078	7,0	1	United States	31735	9,6
2	China	24188	6,5	2	United States	36873	7,0	2	China	26515	8,0
3	Germany	19459	5,2	3	Italy	36678	6,9	3	Italy	25290	7,6
4	United States	18206	4,9	4	Germany	35010	6,6	4	Germany	20833	6,3
5	Netherlands	16826	4,5	5	France	25315	4,8	5	France	17655	5,3
6	France	16773	4,5	6	Japan	21859	4,1	6	Japan	17260	5,2
7	Japan	14975	4,0	7	Netherlands	20236	3,8	7	United Kingdom	10302	3,1
8	Poland	11813	3,2	8	Korea	14212	2,7	8	Spain	8275	2,5
9	Finland	9369	2,5	9	United Kingdom	14147	2,7	9	Poland	7817	2,4
10	Korea	8496	2,3	10	Poland	13601	2,6	10	Korea	7535	2,3

Note: the Rest of the World is dropped from the list of exporters.

Source: WIOD database, author's calculations.

A remarkable finding from Table 2 is that the importance of the United States as an export destination for Russia is much higher than revealed by the traditional trade statistics. In

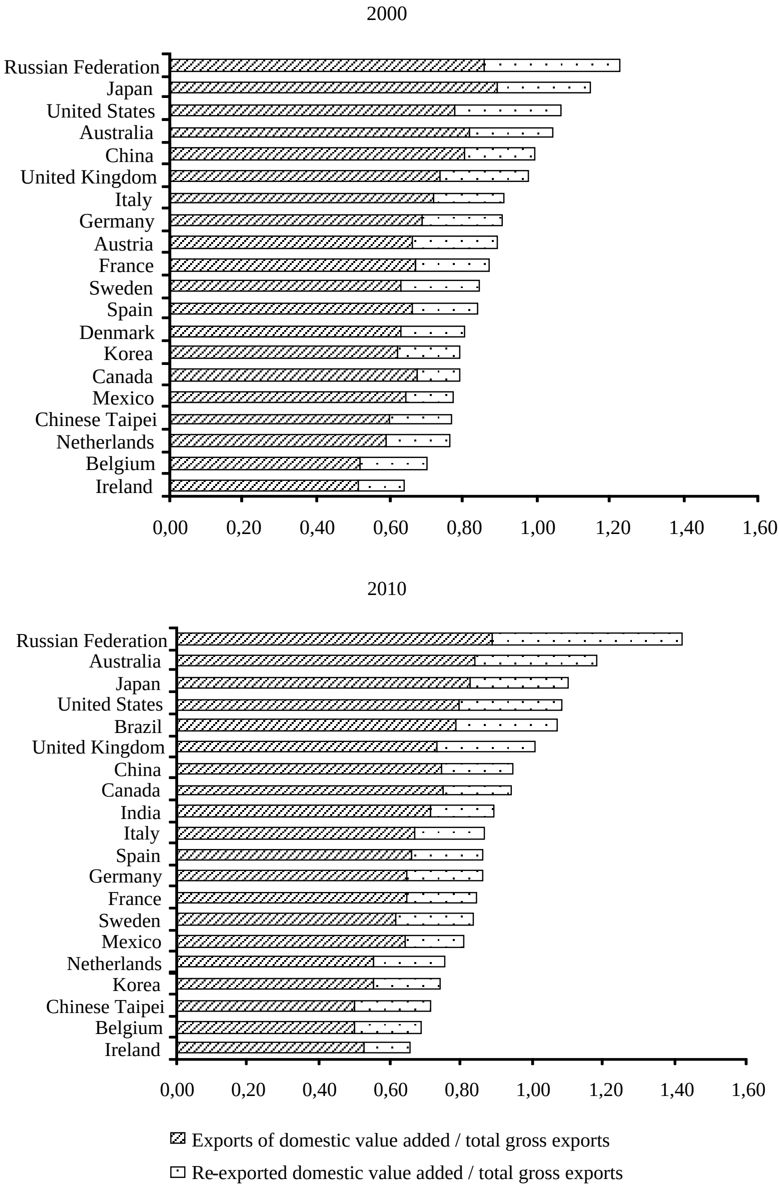
2010, 4,9% of Russia's gross exports were directly sent to the United States, but the share of total exported value added from Russia that eventually ended up in the United States was 9,6% which effectively made the United States the largest consumer of the value added of Russian origin. Similarly, China accounted for 6,5% of Russia's gross exports and 8,0% of its exported value added. The EU members were collectively the largest market for Russia's exports in gross terms (43,4%) and contributed somewhat lower share to the final use of its value added (40,6%). Apparently, the integration into global value chains allowed Russia to serve distant markets via intermediate supply to the closer European neighbours.

4.2. Understanding the basic components of Russia's value added flows

The basic accounting relationship (3) links together two concepts from Tables 1 and 2 – «domestic value added in exports» ($\mathbf{S}'_n[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{bil}]\mathbf{i}$) and «exports of domestic value added» ($\mathbf{S}'_n[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{F}]\mathbf{i}$) – and relates the difference between the two to the domestic value added re-exported by trading partners ($\mathbf{S}'_n[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{tot}]\mathbf{i}$), which is hereafter referred to as the «re-exported value added» for brevity. This term statistically captures cumulative double counted flows of value added that circulate across borders and are largely responsible for the growth of global value chains. It is also known as VS1 [Hummels et al., 1999], or an indicator of «forward participation» in global value chains (various OECD publications), or «downstream component» [UNCTAD, 2013]. Application of the accounting relationship (3) allows one to see which of the two components – exports of value added for final demand or for re-exports – and to what extent is responsible for cumulative value added in global exports that can be attributed to a single country. Figure 2 reports the results as the ratios to gross exports in 2000 and 2010, partly isolating the effect of different years' prices. Top 20 exporters are ranked according to the value added to total gross exports ratios.

It appears that six countries in the WIOD database – the United States, Japan, United Kingdom, Russia, Australia and Brazil – have total domestic value added circulating in global exports that is equal or exceeds their total gross exports. By the way, China is quite close to assume this pattern if it beefs up its downstream value added exports. Note that the United States, Japan and United Kingdom have relatively stable ratios of domestic value added in global exports to total gross exports. Visible changes, if any, e.g. decline in Japan's ratio in 2000–2010 by 0.04, mostly correspond to the respective change in its «final», or «absorbed» component. Meanwhile, Brazil, Australia and Russia experienced an increase in their ratios through 2000–2010, respectively by 0,07, 0,14 and 0,19. The source of the increase was the «re-exported» component which rose by 0,06 in Brazil, by 0,11 in Australia and by 0,16 in Russia. This effectively discerns a group of the upstream suppliers of services in global value chains (United States, Japan, United Kingdom) and a group of the upstream suppliers of natural resources (Russia, Australia, Brazil).

Russia stands out for the extremely high contribution of both value added that ends up in final demand and the value added that is further re-exported, if the domestic gross exports is used as a benchmark. The re-exported component expressed as the ratio to gross exports shows the magnitude and growth rate that is unparalleled in the WIOD database. However, the use of other databases with superior country coverage, e.g. OECD – WTO TiVA database built on the OECD ICIO system, would reveal similar pattern for such resource-rich countries as Norway, Saudi Arabia and Chile.



Note: the Rest of the World is dropped from the list of exporters.

Fig. 2. The basic decomposition of value added flows, ratio to total gross exports
Source: WIOD database, author's calculations.

4.3. The anatomy of Russia's value added in gross exports

Equation (11) offers the itemised decomposition of the bilateral value added flows normalised to the bilateral gross exports. This is particularly useful to see the indirect trade in a country's value added among its trade partners, that is not a direct component of gross exports and cannot be observed in simple versions of value added accounting.

Table 3 relates each component bilateral value added flow to Russia's gross exports to top 20 destinations (ranked as of 2010) in 2000 and 2010. The last two rows summarise the results for total Russia's gross exports. Note that in column F, the values are negative which is required so that all components sum up to gross exports as in equation (11). Yet the sum of the values in columns A through G does not equal 1 because value added is usually somewhat less than total output minus intermediate demand.

Two trends can be clearly discerned: (1) a decline of the relative importance of the direct deliveries of final products to nearly all principal export markets, and (2) significant rise of the indirect trade in Russia's value added among its partners which is in fact double-counted from the value chain perspective. In 2000, only two countries – the United States and United Kingdom – were heavily involved in trading value added that ultimately originated from Russia. In 2010, Germany and Spain joined, each indirectly receiving Russia's value added equivalent to more than 0,9 of the direct gross exports from Russia while Belgium received 1,95! For China, France, Japan, Korea, Sweden, Turkey this measure exceeded 0,5. In 2010, many of the largest Russia's export partners have negative balance of trade in Russia's value added with third countries which explains why they earn higher profile as destinations for Russia's value added when the indirect flows are explicitly accounted for. Italy and the Netherlands are notable exceptions: they indirectly re-export more Russia's value added than they receive.

The last rows in Table 3 confirm that the amount of total Russia's value added that direct partners re-export to third countries equals total Russia's value added that direct partners indirectly receive via third countries. These two components can be summed to cancel each other. This may be treated as partner balance in trading Russia's value added that is zero at the aggregate country level.

Table 3.

Bilateral flows of Russia's domestic value added, decomposed and normalised to gross exports to twenty largest export destinations, an itemised decomposition

Partner	Year	Gross exports, US$ mln (current prices)	Value added flows, ratios to gross exports						
			Domestic, in final products			Domestic, in final and intermediate products (double-counted)			Foreign
			A	B	C	D	E	F	G
Italy	2000	8507	0,03	0,69	0,07	0,00	0,28	–0,20	0,08
	2010	32783	0,01	0,69	0,07	0,01	0,34	–0,20	0,04
China	2000	3008	0,04	0,77	0,07	0,00	0,25	–0,31	0,13
	2010	24188	0,03	0,97	0,10	0,01	0,43	–0,64	0,05

Continues

Partner	Year	Gross exports, US$ mln (current prices)	Value added flows, ratios to gross exports						
			Domestic, in final products			Domestic, in final and intermediate products (double-counted)			Foreign
			A	B	C	D	E	F	G
Germany	2000	16447	0,19	0,49	0,10	0,00	0,34	–0,25	0,09
	2010	19459	0,13	0,61	0,33	0,02	0,71	–0,91	0,05
United States	2000	3844	0,12	1,14	0,57	0,00	0,28	–1,30	0,14
	2010	18206	0,08	1,22	0,45	0,00	0,28	–1,14	0,05
Netherlands	2000	2165	0,03	0,39	0,15	0,00	0,78	–0,50	0,09
	2010	16826	0,02	0,22	0,08	0,00	0,87	–0,29	0,04
France	2000	5304	0,03	0,70	0,15	0,00	0,40	–0,42	0,09
	2010	16773	0,06	0,77	0,22	0,01	0,45	–0,61	0,04
Japan	2000	2915	0,19	0,75	0,18	0,00	0,20	–0,48	0,11
	2010	14975	0,07	0,90	0,18	0,00	0,30	–0,56	0,05
Poland	2000	4069	0,02	0,67	0,04	0,01	0,27	–0,15	0,09
	2010	11813	0,02	0,58	0,07	0,02	0,47	–0,24	0,04
Finland	2000	3263	0,06	0,39	0,02	0,01	0,46	–0,07	0,09
	2010	9369	0,02	0,40	0,04	0,02	0,54	–0,12	0,05
Korea	2000	1761	0,08	0,56	0,08	0,00	0,44	–0,31	0,10
	2010	8496	0,07	0,68	0,14	0,01	0,77	–0,77	0,05
Hungary	2000	2228	0,01	0,61	0,04	0,01	0,37	–0,16	0,09
	2010	6521	0,04	0,51	0,06	0,01	0,49	–0,21	0,04
Spain	2000	2593	0,03	0,73	0,17	0,00	0,41	–0,47	0,09
	2010	6513	0,07	0,86	0,34	0,00	0,59	–0,96	0,04
United Kingdom	2000	1884	0,12	0,90	0,53	0,00	0,50	–1,19	0,10
	2010	5777	0,10	0,96	0,72	0,01	0,66	–1,56	0,05
Sweden	2000	683	0,11	0,68	0,42	0,00	0,82	–1,16	0,09
	2010	5615	0,04	0,28	0,21	0,00	0,95	–0,58	0,04
Greece	2000	1457	0,06	0,70	0,08	0,00	0,26	–0,23	0,09
	2010	5378	0,05	0,75	0,10	0,00	0,23	–0,22	0,04
Lithuania	2000	1378	0,11	0,37	0,03	0,02	0,39	–0,06	0,09
	2010	5119	0,07	0,34	0,07	0,01	0,54	–0,12	0,04
Turkey	2000	2542	0,04	0,80	0,07	0,00	0,18	–0,24	0,11
	2010	4325	0,16	0,70	0,20	0,01	0,32	–0,53	0,06

Continues

Partner	Year	Gross exports, US$ mln (current prices)	Value added flows, ratios to gross exports						
			Domestic, in final products			Domestic, in final and intermediate products (double-counted)			Foreign
			A	B	C	D	E	F	G
Slovak Republic	2000	2174	0,03	0,34	0,02	0,06	0,49	–0,07	0,08
	2010	3596	0,03	0,45	0,07	0,02	0,56	–0,21	0,04
Bulgaria	2000	1674	0,03	0,36	0,01	0,02	0,50	–0,05	0,08
	2010	3338	0,02	0,42	0,04	0,04	0,53	–0,14	0,04
Belgium	2000	717	0,11	0,55	0,51	0,01	1,36	–1,69	0,11
	2010	2973	0,08	0,64	0,46	0,01	1,63	–1,95	0,05
TOTAL (all partners)	2000	98757	0,13	0,60	0,13	0,01	0,37	–0,37	0,10
	2010	371743	0,08	0,65	0,17	0,01	0,52	–0,52	0,05

Note: A – direct, in final products, absorbed by partners;
B – direct, in intermediate products, processed and absorbed by partners;
C – indirect, in intermediate products, processed by third countries and absorbed by partners;
D – reflected, in final and intermediate products (double-counted);
E – re-exported to third countries, in final and intermediate products (double-counted);
F – indirect via third countries, in final and intermediate products (double-counted);
G – foreign, in final and intermediate products.

Russia's partners are ranked according to bilateral gross exports in 2010. The Rest of the World is excluded.

Source: WIOD database, author's calculations.

4.4. Discovering the sectoral origin of value added in Russia's exports

The basic accounting equation in the KN × K dimension $\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{bil}\right] = \left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{F}\right] + \left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot}\right]$ uncovers sector detail behind the aggregated results. Figure 3 shows the contribution of top 10 Russia's sectors to the total domestic value added in exports in 2000 and 2010, and the breakdown discerns the value added for final demand and for re-exports.

Both graphs for 2000 and 2010 show the prevalence in terms of value added in exports of a few sectors that generate trade and transport margins. However, it is «Mining and quarrying» that is clearly responsible for an expansion of Russia's value added flows to the downstream value chain through 2000s. And the re-exported component of the value added created in this sector was growing faster than the finally absorbed one. The manufacturing sectors other than fuel production had their relative value added in exports shrunk through this period.

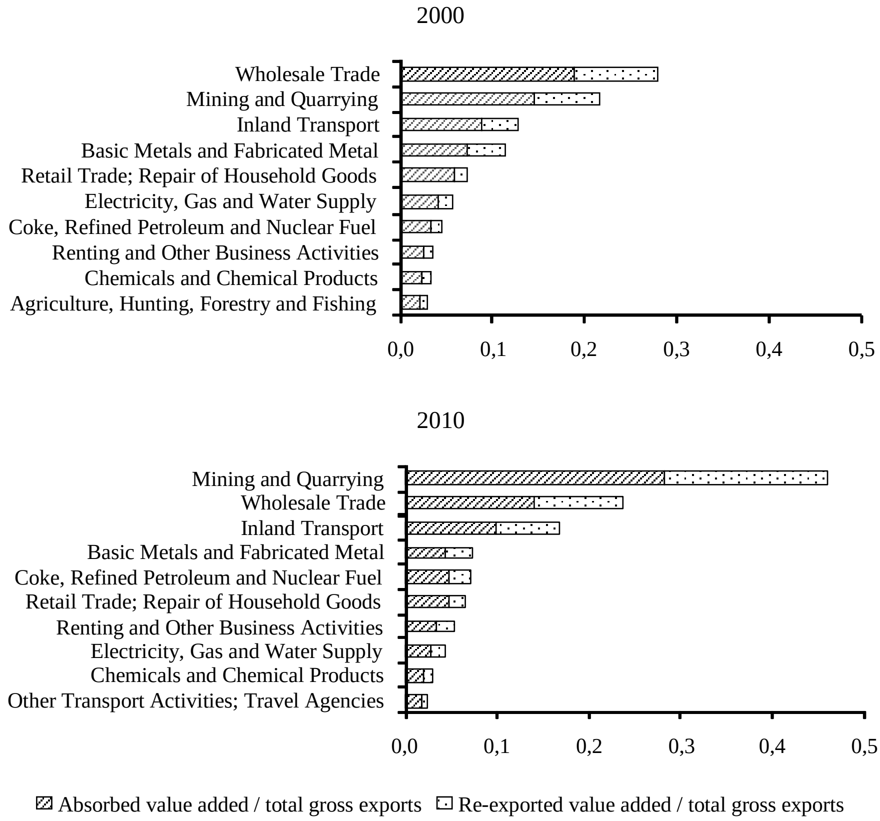


Fig. 3. Russia's domestic value added global flows
from ten largest exporting sectors, a basic decomposition

Source: WIOD database, author's calculations.

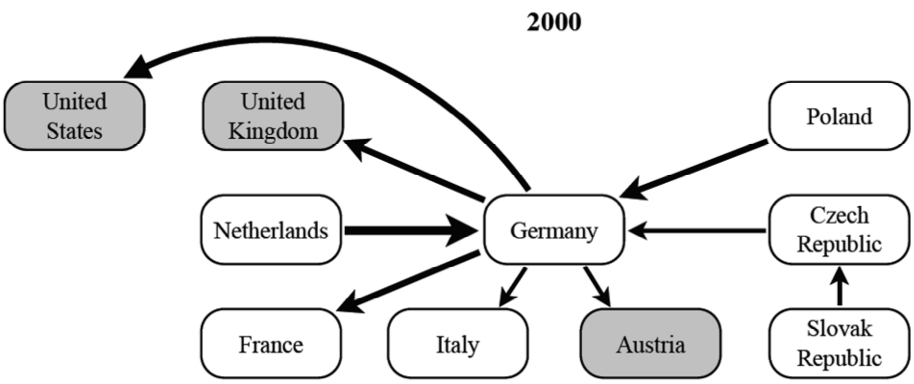
4.5. Quantifying the role of partners and third countries in trading Russia's value added

Let us focus on the single sector that is critical for Russia's export performance, «Mining and quarrying». The application of the equation (10) for the itemised decomposition of the value added originating in this sector in Russia may reveal that the gross export figures disguise the sizable increase in indirect value added flows among Russia's export partners and third countries. Aggregated country-wise in either direction (re-exported from partners to third countries or re-directed from third countries to partners), this value added flow is equal to 0,25 of the total gross exports of «Mining and quarrying» in 2000 and 0,50 in 2010. Many top partners are now actively engaged in indirect imports rather than exports of intermediates containing the value added from Russia's mining and quarrying sector. In 2000, among the top twenty export destinations, four were in fact net indirect importers of the value added of the said origin (i.e. they re-exported less Russia's value added than they indirectly received): Turkey, United Kingdom, China and Austria. In 2010, this status accrued to nine of the top twenty export destinations: China, France, Japan, Germany, the United States, Spain, United Kingdom, Romania and Czech Republic (see [Muradov, 2014] for a graphic illustration).

The matrix representation of the detailed value added components in this paper makes them highly customisable for specific analytical purposes. This is exemplified below in another application not covered in section 3 that will allow one to see how the value added from Russia's «Mining and quarrying» sector is embodied in the back-and forth trade among Russia's partners.

First, the bilateral «foreign value added in trade» matrix in the KN × K dimension $\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$ is modified to include Russia's value added only. The result is a KN × K matrix of trade in Russia's sectoral value added among all K countries. Next, extracting the rows sector-wise gives N K × K matrices that depict the bilateral flows of value added originating in Russia's sector *i* in partner trade. The matrix elements should be normalized, e.g. with respect to sectoral gross exports, for a sensible visualisation.

An example of such visualisation is shown in Fig. 4 that identifies the most important flows of the value added originating in Russia's «Mining and quarrying» sector. If 0,3% of the total gross exports of that sector is chosen as a threshold, the dominant flows in 2000 are confined almost entirely to Europe. Many European countries appeared as the net exporters of value added of the said origin and Germany was a trade hub. Germany also re-exported that value added to the United States. By 2010, more countries shifted to the net importer status while the pattern of the value added flows became more complex. The directions of flows are now more diversified and no single hub may be discerned though the Netherlands rather than Germany appear to be at the centre of this network. Finally, East Asia enters the picture and China emerges as the dominant re-exporter if Russia's value added from mining and quarrying to the United States.



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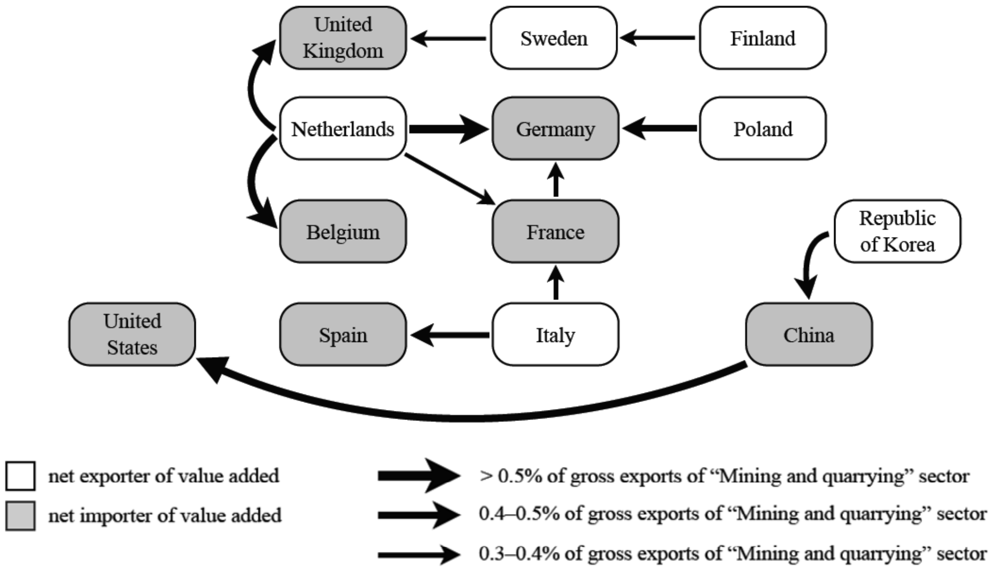

Note: The Rest of the World is excluded.

Fig. 4. Circulation of the value added originating in Russia's «Mining and Quarrying» sector embodied in bilateral gross exports

Source: WIOD database, author's calculations.

4.6. Decomposition at destination: quantifying Russia's value added embodied in domestically consumed and re-exported goods and services at partner side

In this subsection, the quest is for the sectors of partner countries that deliver products where Russia's value added is embodied and for the actual use of those products, i.e. final domestic use or re-exports.

The respective row from equation (22) gives the flow of the value added from «Mining and Quarrying» in Russia to sectors in other countries where it is embodied for further use. In particular, the rows in $\left[\mathbf{V_c L}\overset{\vee}{\mathbf{F}}_{(KN\times KN)}\right]$ and $\left[\mathbf{V_c L}\overset{\vee}{\mathbf{E}}_{tot(KN\times KN)}\right]$ contain estimates of the use of the value added sourced from Russia's «Mining and Quarrying» in the products of partner countries' sectors and their sum is equal to the respective row sum in $\left[\mathbf{V_c L}\overset{\vee}{\mathbf{E}}_{bil(KN\times KN)}\right]$, or the total value added sourced from «Mining and Quarrying». These two terms may be treated as the demand factors for the generation of value added in the sector considered. Meanwhile, $\left[\mathbf{V_c L}\overset{\vee}{\mathbf{Z}}^{*}\right]$ explains the individual sectoral deviations between the amount of the value added that left the origin and the amount that was received at the destination. For example, in 2010, 0,09% of the total value added from Russia's mining and quarrying sector was directly and indirectly sent to partner countries in the form of the products of the construction sector. At the same time, 8,94% of that value added was found embodied in the products of construction for final domestic use worldwide, and 0,10% in the products of construction for re-exports. The difference of 8,95% (the respective entry in $\left[\mathbf{V}_c\check{\mathbf{L}}\mathbf{Z}^*\right]$) flowed from Russia's mining and quarrying to the products of construction indirectly through the products of other sectors in the recipient countries. This means that the use of «Construction» products abroad is a significant factor not directly observed and creates demand for the exports from Russia's «Mining and Quarrying» sector.

Differentiating value added by sector at destination leads to an alternative decomposition of a country's total value added in both direct and indirect exports. The visualisation of results in KN × KN, or [country/sector] × [country/sector] dimension, even for one country, is quite a complex task, so the figures below narrow the focus, again, to Russia's «Mining and Quarrying» exporting sector.

In Fig. 5, the total value added flow from «Mining and Quarrying» is aggregated across all partner countries and disaggregated across recipient sectors. This is to show the importance of the inter-sectoral transfer term: the products of «Mining and Quarrying» appear as the largest carriers of Russia's value added that originates in that sector (55,3%), but the bulk of it (53,5%) is eventually used in the products of other sectors. Figure 6 exemplifies a decomposition of Russia's value added by products of the partner countries' sectors where it is embodied for domestic final use and re-exports. Inter-sectoral term is dropped for a clearer focus on the use of products in partner countries, or, in other words, the ultimate destination of Russia's value added created in the «Mining and Quarrying» sector. This concept is reverse to that in Fig. 3 which discerned the sectoral origin of value added in exports.

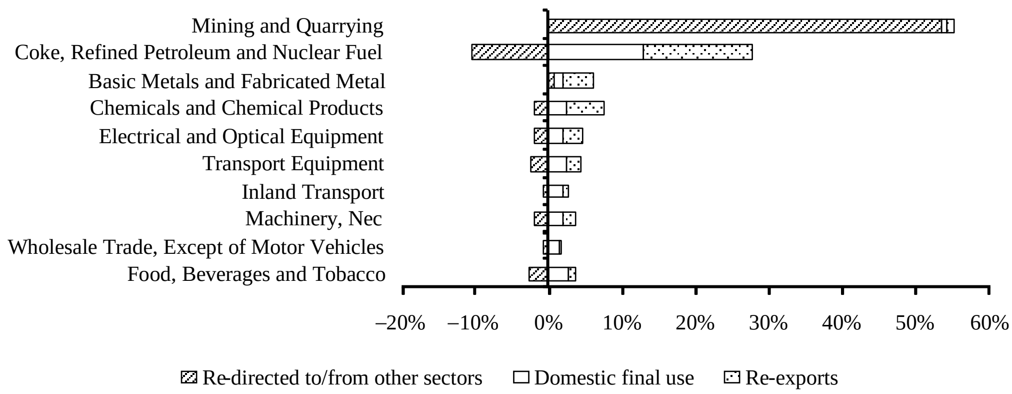

Fig. 5. Decomposition of the value added from Russia's «Mining and Quarrying» sector in gross exports according to the products of partner countries' sectors where it is embodied, including the inter-sectoral transfer term, 2010 (percentage of total)

Source: WIOD database, author's calculations.

As might be expected, the products of «Coke, Refined Petroleum and Nuclear Fuel» sector for both domestic use and exports appear as the principal absorbers of the value added created in «Mining and Quarrying». Other products contributing to the demand for Russia's value added from «Mining and Quarrying» include those of «Construction» and «Electricity, Gas and Water Supply» for domestic use, «Basic Metals and Fabricated Metal», «Chemicals and Chemical Products» for exports. Interestingly, the top 10 list also includes such entries as the products of «Food, Beverages and Tobacco» and «Public Administration and Defence, Compulsory Social Security». One can therefore conclude that in 2010, domestic worldwide use of public administration and defence services worth a dollar helped generate nearly 3 cents of value added in Russia's mining and quarrying.

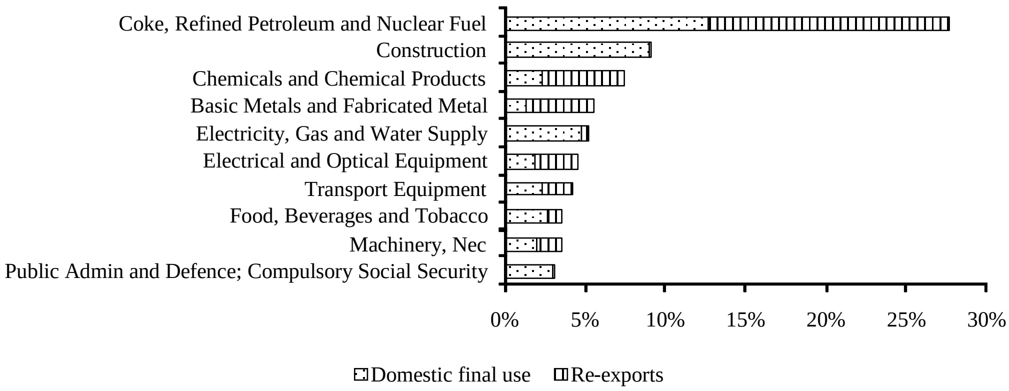

Fig. 6. Decomposition of the value added from Russia's «Mining and Quarrying» sector in gross exports according to the products of partner countries' sectors where it is embodied for final use or re-exports, 2010 (percentage of total)

Source: WIOD database, author's calculations.

An additional partner country breakdown (not shown here) reveals that European economies largely use Russia's value added from «Mining and Quarrying» to produce and export fuels. Contrary to European countries, China mostly uses that value added of the Russian origin for domestic construction and exports of electrical and optical equipment. The United States use it for domestically consumed fuels but also for public administration and defence. In other words, every dollar spent in the United States for public administration and defence services generates 0,9 cents of value added in Russia's mining and quarrying sector. This is an intrinsic implication of global value chains that link two seemingly unrelated sectors in two distant countries

5. Conclusion

Gross exports accounting frameworks build on the recent progress in developing global input-output databases to identify the origin and destination of value added behind conventional trade statistics. The decomposition of gross trade flows utilises common value added concepts, but the exact method may vary. The starting point may be direct bilateral gross exports disentangled into additive value added components, and the results should be useful for trade policy analysis. Or it may be direct and indirect (cumulative) bilateral flows of value added, from the origin to the destination. The results then are thought to be more relevant for global value chain analysis. It is this type of frameworks that this paper intends to generalise and elaborate.

At the core of the technical discussion is an elegant and simple way to derive a basic decomposition of the cumulative value added flows that attribute each flow to the country and sector of origin and to the country and sector of destination. Two basic components include value added that «ends up», or is finally absorbed, in partner country and value added that only «lands» in partner country to be further re-exported. The first component is in fact a part of a country's GDP that is absorbed (finally used) overseas and can be treated as the ultimate external demand factor that contributes to the GDP. The second component corresponds to the double counted flows of value added that are responsible for inflating gross exports figures and may be treated as an intermediate demand factor. Both components can not be directly observed in gross trade statistics.

There are various ways to split these basic components and obtain more detailed indicators or aggregate those across sectors or countries or both. The matrix representation appears to be highly customisable and adaptable to matrix computation software. The formulations proposed yield results that are mostly identical to those of [Stehrer, 2013] who built his work on [Koopman et al., 2012]. The discussion has also led to the derivation of two matrices of the inter-sectoral transfer of value added for which, respectively, the exporting country and partner country is responsible. These matrices, to the author's knowledge, didn't explicitly feature previous studies. Excluding these two matrices or respective matrix elements from the value added accounting equations at the bilateral sectoral level will make them incomplete. The basic form of the derived accounting relationship is also used to prove in a quick and efficient way that the total trade balances are equal in gross and value added terms.

Applied to real data from the WIOD database, the proposed formulations uncover a great deal of detail intrinsic to the expanding global value chains. Gross trade statistics hide a multitude of indirect linkages that shape countries' export performance. Indirect linkages show how production in one country responds to final or intermediate demand in another country while visible link may appear weak or may not exist at all. Russia seems to be a very good example to test the significance of such indirect links. Exported value added of Russian origin – primarily from the mining and quarrying sector – is then repeatedly used in the downstream value chain, and to a higher extent than for any other country in the WIOD database. This has little effect on the total gross exports or total value added satisfying foreign final demand, but affects Russia's bilateral relations with trade partners. Indirect links shed light on the importance of some partners. For example, flows of Russia's value added that end up in the United States' final domestic consumption are largely not governed by direct trade policies, as those are mostly indirect flows.

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