

Contents

1	Introduction	<i>page</i> 1
	About This Book	1
Part I	Technical Basics	5
2	A Primer on Probability	7
2.1	Probability and Measure	7
2.2	Filtrations and the Flow of Information	10
2.3	Conditional Probability and Independence	11
2.4	Random Variables and Stochastic Processes	14
2.5	Moments of Random Variables	17
2.6	Characteristic Function and <i>Fourier</i> -Transform	20
2.7	Further Reading	24
2.8	Problems	24
3	Vector Spaces	26
3.1	Real Vector Spaces	26
3.2	Dual Vector Space and Inner Product	30
3.3	Dimensionality, Basis, and Subspaces	34
3.4	Functionals and Operators	37
3.5	Adjoint and Inverse Operators	43
3.6	Eigenvalue Problems	45
3.7	Linear Algebra	47
3.8	Vector Differential Calculus	53
3.9	Multivariate Normal Distribution	55
3.10	Further Reading	57
3.11	Problems	58
4	Utility Theory	60
4.1	Lotteries	60
4.2	Preference Relations and Expected Utility	61
4.3	Risk Aversion	63
4.4	Measures of Risk Aversion	65
4.5	Certainty Equivalent and Risk Premium	66
4.6	Classes of Utility Functions	68

4.7	Constrained Optimization	70
4.8	Further Reading	74
4.9	Problems	74

Part II Financial Markets and Portfolio Theory 77

5 Architecture of Financial Markets 79

5.1	The <i>Arrow-Debreu</i> -World	79
5.2	The Portfolio Selection Problem	81
5.3	Preference-Free Results	83
5.4	<i>Pareto</i> -Optimal Allocation and the Representative Agent	89
5.5	Market Completeness and Replicating Portfolios	92
5.6	Martingale Measures and Duality	96
5.7	Further Reading	97
5.8	Problems	98

6 Modern Portfolio Theory 100

6.1	The <i>Gaussian</i> Framework	100
6.2	Mean-Variance Analysis	104
6.3	The Minimum Variance Portfolio	109
6.4	Variance Efficient Portfolios	111
6.5	Optimal Portfolios and Diversification	113
6.6	Tobin's Separation Theorem and the Market Portfolio	115
6.7	Further Reading	118
6.8	Problems	119

7 CAPM and APT 120

7.1	Empirical Problems with MPT	120
7.2	The Capital Asset Pricing Model (CAPM)	121
7.3	Estimating Betas from Market Data	125
7.4	Statistical Issues of Regression Analysis and Inference	129
7.5	The Arbitrage Pricing Theory (APT)	135
7.6	Comparing CAPM and APT	140
7.7	Further Reading	141
7.8	Problems	141

8 Portfolio Performance and Management 143

8.1	Portfolio Performance Statistics	143
8.2	Money Management and <i>Kelly</i> -Criterion	145
8.3	Adjusting for Individual Market Views	150

8.4	Further Reading	154
8.5	Problems	154
9	Financial Economics	156
9.1	The Rational Valuation Principle	156
9.2	Stock Price Bubbles	160
9.3	Shiller's Volatility Puzzle	164
9.4	Stochastic Discount Factor Models	166
9.5	C-CAPM and <i>Hansen–Jagannathan</i> -Bounds	169
9.6	The Equity Premium Puzzle	172
9.7	The <i>Campbell–Cochrane</i> -Model	175
9.8	Further Reading	179
9.9	Problems	180
10	Behavioral Finance	181
10.1	The Efficient Market Hypothesis	181
10.2	Beyond Rationality	185
10.3	Prospect Theory	188
10.4	Cumulative Prospect Theory (CPT)	191
10.5	CPT and the Equity Premium Puzzle	194
10.6	The Price Momentum Effect	197
10.7	Unifying CPT and Modern Portfolio Theory	199
10.8	Further Reading	205
10.9	Problems	205
Part III	Derivatives	207
11	Forwards, Futures, and Options	209
11.1	Forward and Future Contracts	209
11.2	Bank Account and Forward Price	210
11.3	Options	213
11.4	Compound Positions and Option Strategies	216
11.5	Arbitrage Bounds on Options	219
11.6	Further Reading	220
11.7	Problems	220
12	The Binomial Model	222
12.1	The Coin Flip Universe	222
12.2	The Multi-Period Binomial Model	225
12.3	Valuating a European Call in the Binomial Model	228
12.4	Backward Valuation and American Options	232

12.5	Stopping Times and <i>Snell</i> -Envelope	235
12.6	Path Dependent Options	240
12.7	The <i>Black–Scholes</i> -Limit of the Binomial Model	243
12.8	Further Reading	246
12.9	Problems	247
13	The <i>Black–Scholes</i>-Theory	249
13.1	Geometric <i>Brownian</i> Motion and Itô's Lemma	249
13.2	The <i>Black–Scholes</i> -Equation	253
13.3	Dirac's δ -Function and Tempered Distributions	256
13.4	The Fundamental Solution	260
13.5	Binary and Plain Vanilla Option Prices	264
13.6	Simple Extensions of the <i>Black–Scholes</i> -Model	267
13.7	Discrete Dividend Payments	270
13.8	American Exercise Right	274
13.9	Discrete Hedging and the Greeks	277
13.10	Transaction Costs	283
13.11	Merton's Firm Value Model	287
13.12	Further Reading	289
13.13	Problems	290
14	Exotics in the <i>Black–Scholes</i>-Model	291
14.1	Finite Difference Methods	291
14.2	Numerical Valuation and Coding	298
14.3	Weak Path Dependence and Early Exercise	301
14.4	Girsanov's Theorem	303
14.5	The <i>Feynman–Kac</i> -Formula	306
14.6	Monte Carlo Simulation	310
14.7	Strongly Path Dependent Contracts	312
14.8	Valuating American Contracts with Monte Carlo	317
14.9	Further Reading	323
14.10	Problems	323
15	Deterministic Volatility	326
15.1	The Term Structure of Volatility	326
15.2	GARCH-Models	328
15.3	Duan's Option Pricing Model	332
15.4	Local Volatility and the <i>Dupire</i> -Equation	334
15.5	Implied Volatility and Most Likely Path	338
15.6	Skew-Based Parametric Representation of the Volatility Surface	343
15.7	<i>Brownian</i> Bridge and GARCH-Parametrization	345
15.8	Further Reading	351
15.9	Problems	351

16 Stochastic Volatility	353
16.1 The Consequence of Stochastic Volatility	353
16.2 Characteristic Functions and the Generalized <i>Fourier</i> -Transform	355
16.3 The Pricing Formula in <i>Fourier</i> -Space	358
16.4 The <i>Heston–Nandi</i> GARCH-Model	362
16.5 The <i>Heston</i> -Model	365
16.6 Inverting the <i>Fourier</i> -Transform	370
16.7 Implied Volatility in the SABR-Model	373
16.8 Further Reading	377
16.9 Problems	378
17 Processes with Jumps	379
17.1 Càdlàg Processes, Local-, and Semimartingales	379
17.2 Simple and Compound <i>Poisson</i> -Process	381
17.3 GARCH-Models with Conditional Jump Dynamics	385
17.4 Merton's Jump-Diffusion Model	389
17.5 Barrier Options and the Reflection Principle	393
17.6 <i>Lévy</i> -Processes	397
17.7 Subordination of <i>Brownian</i> motion	402
17.8 The <i>Esscher</i> -Transform	406
17.9 Combining Jumps and Stochastic Volatility	410
17.10 Further Reading	412
17.11 Problems	412
Part IV	415
The Fixed-Income World	415
18 Basic Fixed-Income Instruments	417
18.1 Bonds and Forward Rate Agreements	417
18.2 LIBOR and Floating Rate Notes	421
18.3 Day-Count Conventions and Accrued Interest	422
18.4 Yield Measures and Yield Curve Construction	425
18.5 Duration and Convexity	430
18.6 Forward Curve and Bootstrapping	433
18.7 Interest Rate Swaps	436
18.8 Further Reading	440
18.9 Problems	440
19 Plain Vanilla Fixed-Income Derivatives	442
19.1 The <i>T</i> -Forward Measure	442
19.2 The <i>Black-76</i> -Model	445
19.3 Caps and Floors	447

19.4	Swaptions and the Annuity Measure	449
19.5	Eurodollar Futures	452
19.6	Further Reading	453
19.7	Problems	454
20	Term Structure Models	455
20.1	A Term Structure Toy Model	455
20.2	Yield Curve Fitting	458
20.3	Mean Reversion and the <i>Vasicek</i> -Model	460
20.4	Bond Option Pricing and the <i>Jamshidian</i> -Decomposition	463
20.5	Affine Term Structure Models	465
20.6	The <i>Heath–Jarrow–Morton</i> -Framework	469
20.7	Multi-Factor HJM and Historical Volatility	475
20.8	Further Reading	481
20.9	Problems	481
21	The LIBOR Market Model	483
21.1	The Transition from HJM to Market Models	483
21.2	The Change-of-Numéraire Toolkit	487
21.3	Calibration to Caplet Volatilities	491
21.4	Parametric Correlation Matrices	494
21.5	Calibrating Correlations and the Swap Market Model	498
21.6	Pricing Exotics in the LMM	502
21.7	Further Reading	506
21.8	Problems	506
A	Complex Analysis	509
A.1	Introduction to Complex Numbers	509
A.2	Complex Functions and Derivatives	513
A.3	Complex Integration	515
A.4	The Residue Theorem	519
B	Solutions to Problems	525
	<i>References</i>	573
	<i>Index</i>	583